VALUATION FORMULAS

Given…

$$PV = \text{present value of one or more future cash flows}$$

$$n = \text{number of periods}$$

$$FV_n = \text{face or nominal value of cash 'n' periods from now}$$

$$r = \text{per-period rate of return}$$

$$\mathrm{PVA_n} = \text{present value of an annuity whose duration is 'n' periods}$$

Future Value Formulas

The value one period from now of an amount invested at rate *r*:

$$\begin{aligned} FV_1 &= PV + PV \times r \\ &= PV \times (1+r) \end{aligned}$$

The value two periods from now of an amount invested at the per-period rate *r*:

$$\begin{aligned} FV_2 &= FV_1 + FV_1 \times r \\ &= FV_1 \times (1+r) \\ &= PV \times (1+r) \times (1+r) \\ &= PV \times (1+r)^2 \end{aligned}$$

The value *n* periods from now of an amount invested at the per-period rate *r*:

$$FV_n = PV \times (1+r)^n$$

NOTE: This last formula subsumes the equations for FV_1 and FV_2.

Compounding

Sometimes interest rates are “compounded” – that is, they are paid more than once a year. Semi-annual compounding entails dividing the stated interest in half but then paying it twice a year; quarterly compounding entails dividing the stated rated by four but paying that every three months… and so on. There is no limit to how often rates can be compounded; they can be compounded daily, hourly – even *continuously*. Compounding increases the total amount earned/paid, though the effect of compounding more and more frequently does reach a limit (specifically, continuous compounding). Compounding is seen most typically with personal savings accounts and with bonds.

Our existing FV/PV formula covers compounding. One simply needs to make sure that the annual rate is converted to the per-period rate and that *n* reflects the comparable number of periods. For instance, with monthly compounding, set…

$$r = \frac{r_{annual}}{12}$$

$$n = yrs \times 12$$

$$and...$$

$$FV_n = PV \times (1+r)^n \text{ is still valid}$$

The general formula for converting a compounded rate to its effective rate is:

$$Effective\ annual\ rate = \left(1 + \frac{r_{annual}}{n}\right)^n$$

where....

$$n = number\ of\ compounding\ periods\ per\ annum$$

For continuous compounding, the formula is:

$$Effective\ annual\ rate\ from\ c.c. = e^r - 1$$

By way of example, 5% compounded....

$$semi-annually \equiv 5.0625\%$$

$$quarterly \equiv 5.0945\%$$

$$monthly \equiv 5.1162\%$$

$$daily \equiv 5.1267\%$$

$$continuously \equiv 5.1271\%$$

Present Value Formulas

Since our formula for FV_n is an all-purpose formula that works for *any* number of periods, rearranging the terms gives us an all-purpose formula for the **present value of a *single* payment *n* periods in the future:**

since...

$$FV_n = PV \times (1+r)^n$$

$$PV = \frac{FV_n}{(1+r)^n}$$

$$= FV_n \times \frac{1}{(1+r)^n}$$

The present value of an *infinite* number of equal payments, each made at the end of the appropriate period – i.e., the present value of a *perpetuity* - is:

$$PV = \frac{pmt}{1+r} + \frac{pmt}{(1+r)^2} + \frac{pmt}{(1+r)^3} + \ldots + \frac{pmt}{(1+r)^\infty}$$

$$PV \times (1+r) = pmt + \frac{pmt}{(1+r)} + \frac{pmt}{(1+r)^2} + \frac{pmt}{(1+r)^3} + \ldots + \frac{pmt}{(1+r)^\infty}$$

subtracting the first line from the second we get...

$$PV \times (1+r) - PV = pmt$$

$$PV \times r = pmt$$

$$PV = \frac{pmt}{r}$$

Present Value Formulas (cont'd)

An *annuity* is a stream of equal payments made or received over a pre-defined length of time. An annuity which lasts for *n* periods is equivalent to a perpetuity begun now *minus* a perpetuity begun in *n* periods. To find the value of an annuity we must discount that second, subtracted perpetuity to its value *today*, which we can do with our standard present value formula. Thus, **the present value of an annuity is**:

$$PVA_n = \frac{pmt}{r} - \frac{\frac{pmt}{r}}{(1+r)^n}$$

$$= \frac{pmt}{r} \times \left(1 - \frac{1}{(1+r)^n}\right)$$

$$= pmt \times \frac{1 - \frac{1}{(1+r)^n}}{r}$$

The Gordon Model

One way of valuing a stock is to say that that its worth lies solely in the dividends it pays. Were an investor to analyze a stock and conclude that the stock will pay the same dividend year after year, that investor could value the stock by viewing it as a simple perpetuity. (Ignoring special situations like bankruptcy, companies are "going concerns", and, for all practical purposes, are assumed to exist *ad infinitum.*)

The difference between a perpetuity and a stock, however, is that there is generally *some* level of growth in dividend payments. Further, for a large, mature company – such as many companies within the Fortune 500 – it is reasonable to assume that such growth has by and large reached a plateau. This kind of stock is termed a "constant-growth stock", and the appropriate way to value such a stock is with **The Gordon Model:**

$$S = \frac{Div_0 \times (1+g)}{\hat{k} - g}$$

$$= \frac{Div_1}{\hat{k} - g}$$

where $\ldots S = \text{share price}$

$$Div_t = \text{dividend per share at time } t$$

$$g = \text{per-period dividend growth rate}$$

$$\hat{k} = \text{total rate of return 'demanded' by investors}$$

Compound Annual Growth Rate (CAGR)

So far we have concerned ourselves with two scenarios:

1. We know the present value of an investment and the applicable rate of return, and we need to calculate the future value of that investment.

2. We know the future value of a cash flow and the appropriate discount rate, and we need to determine its present value.

However, what if it is neither PV nor FV, but *r* itself that we need to determine? Assuming that we know PV and FV we can simply rearrange our basic equation and then solve for r. Since….

$$FV_n = PV(1+r)^n$$

$$(1+r)^n = \frac{FV_n}{PV}$$

$$r = \left(\frac{FV_n}{PV}\right)^{1/n} - 1$$

When using this equation, note that *n* does *not* have to be a whole number.

To calculate the continuously compounded annual rate of return, rather than the simple annual rate of return, the formula is:

$$r_{c.c.} = \frac{\ln\left(\frac{FV_t}{PV}\right)}{t}$$

where…

t=length of time in years.

By way of example, if an investor puts \$5 million of equity into a hotel development project, and, 6.5 years later, having received no dividends, sells her interest for \$8.3 million, the return on her investment was:

$$CAGR_{simple} = \left(\frac{8.3}{5.0}\right)^{1/6.5} - 1 = 8.11\%$$

$$CAGR_{c.c.} = \ln\left(\frac{8.3}{5.0}\right) \Big/ 6.5 = 7.80\%$$