LECTURE 10
Interest Rate Forwards and Options
Chapter 12

Add-on Interest Method:
Consider a Eurodollar time deposit that pays \$1 in 90 days (3 months). At a current rate of 8 %, this Eurodollar has a value of $\$1/[(1+0.08)^{(90/360)}] = \$1/(1.08)^{.25} = \$0.9804$. Suppose that a forward contract has been created on this Eurodollar and that contract is expiring right now. The payoff of a long position would be \$0.9804 minus the forward price agreed to when the contract initiated. Forward contracts on interest rates and forward contracts on Eurodollars are related but different contracts.

Discount Interest Method (Treasury Bill):
For a rate of 8%, a Treasury bill price would be $\$1[1 – 0.08(90/360)] = \$1(1-.02)=\$0.98$.

- A long position in a forward contract on a Treasury bill would pay off 1 minus 0.08(90/360) minus the forward price.
- A short position in a forward contract on the rate would pay off the forward rate minus 0.08 times 90/360.
  - long position in a forward contract = short position in a forward contract on a rate with the forward rate based on one minus the forward price.
  - Derivatives on interest rates are essentially the same as derivatives on bonds.

Primary types of instruments:

1. Forward Rate Agreements (FRA): A forward contract in which the two parties agree to make interest payments to each other at future dates. One party makes a payment at a rate agreed to in advance. The other party makes a payment at a rate to be determined later.
2. Interest Rate Option: One party pays the other a premium today and receives the right to :
   a. make a known interest payment (IR Call) and receive unknown interest payment at a future date; or
   b. receive a known interest payment (IR Put) and make an unknown interest payment at a future date
3. Swaption: The buyer of a swaption in which the underlying asset is an interest rate swap pays a premium and receives the right to enter into a swap to
   a. pay the fixed rate and receive the floating rate; or
   b. pay the floating rate and receive the fixed rate
4. Forward Swap: A forward contract to enter into a swap today.

1. Forward Rate Agreements (FRA):

FRA is similar to any type of forward contract, but the payoff is based on an interest rate rather than a price of an asset.

Payoff (based on LIBOR)
Payoff = (Notional Amount) [(L – Agreed upon price) (d/360)) / (1+L (d/360)]

Example:
XYZ Bank is taking a long position (receive floating rate) in a FRA based on 90-day LIBOR that expires in 30 days. The notional amount is $20mm. Let the rate agreed upon by the two parties be 5 percent. Thus, in 30 days, the payoff to the holder of the long position (XYZ Bank) will be $20,000 * [(L – 0.05 (90/365)/(1+L(90/360)]

Suppose L at expiration 4.0% then
$20,000 * [(0.04 – 0.05 (90/365)/(1+0.04(90/360)]=-$49,505
That means the party who is long has to pay $49,505 to the party who is short

If L at expiration is 6.0% then:
$20,000 * [(0.06 – 0.05 (90/365)/(1+0.06(90/360)]=$49,261
That means the party who is long receives $49,261 from the party who is short

So, when LIBOR (L) is higher than 5.0%, the holder of the long position receives a positive payoff to be paid by the counterparty

INVESTOPEDIA EXAMPLE: Calculation of Payment at Expiration of FRA
Let's set up the transaction:

- Dealer quotes a rate of 4% on this instrument and end user agrees. He is hoping that rates will increase.
- Expiration is in 90 days.
- The notional amount is $ 5 million.
- The underlying interest rate is the 180 LIBOR time deposit.
- In 90 days the 180-day LIBOR is at 5%. That 5% interest will be paid 180 days later.

So: $5{,}000{,}000 \times \dfrac{(0.05 - 0.04)(180/360)}{1 + 0.05(180/360)} = \$\,47{,}600$

Because rates increased, the long party or the end user will receive $47,600 from the short party or the dealer.

If the rates were to decrease, the long party or the end user would have to pony up a payment that would be the difference between the quoted rate and the 180-day LIBOR rate

Read more: Characteristics of Forward Rate Agreements (FRAs) - CFA Level 1 | Investopedia http://www.investopedia.com/exam-guide/cfa-level-1/derivatives/characteristics-fras-forward-rate-agreements.asp#ixzz4GeiGYzAM

Valuing FRA's

INPUT:			
Description	**Symbol**		
Notional Amount		$ 20,000,000	
LIBOR (spot of h) lets call La	**L(h)**	**5.50%**	
LIBOR (Referance Days)	m	90	days
LIBOR (Days FRA exp)	h	30	days
LIBOR (spot of h+m) lets call Lb	**L(m+h)**	**5.14%**	
LIBOR (Days m+h)	m+h	120	days
Basis		360	days
Days into its life	g	20	days
Days remaining (h-g)	h-g	10	days
Days of Reference + Remaining	h+m-g	100	days
LIBOR (spot of h-g) lets call Lc	**L(h-g)**	**5.25%**	
LIBOR (spot of h+m-g) lets call Ld	**L(h+m-g)**	**5.10%**	

1. Determining the Forward Rate

STEP 1 (solving for the rate of FRA):		**Numer (N)**	**Denom (D)**	**F**
Formula		**1+(Lb*((m+h)/360)**	**1+(La*(h/360)**	**'D-1)*(360/m)**
Finding Forward Rate	F	1.017133333	1.004583333	5.00%

2. Valuing a Forward Rate Agreement during life

STEP 2 (valuing FRA durinng its life)		**Adj for Days Remain (A)**	**FRA Rate Calc**		**VFRA**
Formula		**1/(1+Lc*((h-g)/360))**	**Num =(1+(step1*(m/360)** **Dem = (1+(Ld*(h+m-g)/360)**		
Valying FRA	VFRA	0.99854379	1.012492742		**$3,886.65**
			1.014166667		
		N/D=	0.998349458		

Hedging Example:
A firm is planning to borrow $20 million in 30 days at 90-day LIBOR plus 100 bps (Spread). The loan will be paid back with principal and interest 90 days later. Concerned about possibility of rising interest rates (LIBOR), the firm would like to lock in the rate it pays by going long an FRA. The rate on 30-day FRAs based on 90-day LIBOR is 5%. Interest on the loan and the FRA is based on factor 90/360. The outcomes for range of LIBORs are shown as follows:

Nominal Val	$ 20,000,000
Spread =	1.00%
Cur. LIBOR=	5.00%

LIBOR on Day 30 (%)	FRA PAYOFF ON DAY 30 ($)	FRA PAYOFF COMPOUND TO DAY 120 ($)	AMOUNT DUE ON LOAN ON DAY 120 ($)	TOTAL AMOUNT PAID ON DAY 120 ($)	EFFECTIVE RATE ON LOAN (%) based on 365 days	EFFECTIVE RATE WITHOUT FRA (%)
1.00%	$ (199,501)	$ (200,000)	$ 20,100,000	$ 20,300,000	6.22%	2.04%
1.50%	$ (174,346)	$ (175,000)	$ 20,125,000	$ 20,300,000	6.22%	2.56%
2.00%	$ (149,254)	$ (150,000)	$ 20,150,000	$ 20,300,000	6.22%	3.08%
2.50%	$ (124,224)	$ (125,000)	$ 20,175,000	$ 20,300,000	6.22%	3.60%
3.00%	$ (99,256)	$ (100,000)	$ 20,200,000	$ 20,300,000	6.22%	4.12%
3.50%	$ (74,349)	$ (75,000)	$ 20,225,000	$ 20,300,000	6.22%	4.64%
4.00%	$ (49,505)	$ (50,000)	$ 20,250,000	$ 20,300,000	6.22%	5.17%
4.50%	$ (24,722)	$ (25,000)	$ 20,275,000	$ 20,300,000	6.22%	5.69%
5.00%	$ (0)	$ (0)	$ 20,300,000	$ 20,300,000	6.22%	6.22%
5.50%	$ 24,661	$ 25,000	$ 20,325,000	$ 20,300,000	6.22%	6.76%
6.00%	$ 49,261	$ 50,000	$ 20,350,000	$ 20,300,000	6.22%	7.29%
6.50%	$ 73,801	$ 75,000	$ 20,375,000	$ 20,300,000	6.22%	7.82%
7.00%	$ 98,280	$ 100,000	$ 20,400,000	$ 20,300,000	6.22%	8.36%
7.50%	$ 122,699	$ 125,000	$ 20,425,000	$ 20,300,000	6.22%	8.90%
8.00%	$ 147,059	$ 150,000	$ 20,450,000	$ 20,300,000	6.22%	9.44%
8.50%	$ 171,359	$ 175,000	$ 20,475,000	$ 20,300,000	6.22%	9.99%
9.00%	$ 195,599	$ 200,000	$ 20,500,000	$ 20,300,000	6.22%	10.53%

Assuming LIBOR moves to 6.0% from 5.0% then on the $20,000 loan the 3 month interest payment will be L+1.0% or 6.0%+1.0%=7.0% annual or 3 months .25x.07x$20,000 = $350,000. Including principal (P+I) the payment will be $20,350,000. Including the FRA for hedging the payment will be $20,300,000 (subtracting $50,000) – thus the firm borrowed $20 million and 90 days later paid back $20,300,000. The effective rate is 6.22% calculated ($20.3mm/$20mm)$^{(365/90)}$ Have the firm not done the FRA, it would have paid back $20,350,000 or 7.29% effective rate calculated ($20.35mm/$20mm)$^{(365/90)}$.

2 Interest Rate Options (IRO)

The difference between FRA and Interest Rate Option (IRO) is that FRAs are firm committed where the IROs are optional. IROs have a set exercise rate or strike rate rather than an exercise price or strike price (options have covered earlier). IROs are more of a European option type nature (exercised at expiration date).

As with swaps and FRAs, IROs are based on a given notional amount on which the interest is calculated. Using X as the exercised interest rate and m-day LIBOR as the underlying asset, the payoff of an interest rate call and put are as follows:

Call IRO:
(Notional Amount) [Max (0, LIBOR – X) (m/360)]

Put IRO
(Notional Amount) [Max (0, X – LIBOR) (m/360)]

Example:
Consider interest rate Call and Put options with notional amounts of $20 million expiring in 30 days; an underlying of 90-day LIBOR; exercise rates of 5%. Let us look at how the payoffs are calculated. Suppose that at expiration, LIBOR drops 1%. The Call Payoff would be:

For Call $20,000,000 * [Max (0,0.01 – 0.05) (90/360)] = 0
For Put $20,000,000 * [Max (0, 0.05 – 0.01) (90/360)] = $200,000

Pricing & Valuation IROs
Using Black-Sholes lets use the same example from the FRAs given the following information:

Example:
F = 0.0520 obtain by ln [(1+0.0514 (120/360)/(1+0.05(30/360)] (365/90)=0.0520
X=0.05 (exercise LIBOR rate)
r = 0.0506 obtain by $1[1+ $0.05^{(30/360)}$] = 1.0041667 to Log and mult 365/30 = 0.0506
σ = 0.3
T = 0.0822
M = 90

Black Scholes results:
d1 = 0.4990
d2 = 0.4130
N (d1) = 0.6915
N (d2) = 0.6591
$e^{-F(m/365)} = e^{-0.0506(0.0822)}$
C = 0.00295244
Contract Price = $20,000,000 (90/360) (0.00295244 = $14,762

3 Interest Rate Swaptions and Forward Rates

Example:
Consider the following situation. A company called MPK Resources is considering the possibility that it will need to engage in an interest rate swap two years from now with a notional amount of $10 million. It expects that the swap would be a three-year, pay fixed, receive floating swap. The firm is concerned about rising interest rates over the next two years that would force it to pay a higher fixed rate if it entered into a swap at that time. It thus decides to purchase a two-year European-style payer swaption where the underlying is a three-year, pay-fixed, receive-floating swap. Naturally, the underlying swap should be identical to the one MPK expects to take out in two years. MPK specifies an exercise rate of 11.5%. MPK pays a premium up front, the amount of which we do not need to know at this point. To keep the illustration as simple as possible, assume that the underlying swap calls for annual interest payments.

Now let us consider what happens when the swaption expires in two years. At that time, we observe a term structure of LIBOR as follows:

SWAPTIONS

Term Days	Rate	Discount Bond Price	R
360	12.00%	0.89285714	12.000%
720	13.28%	0.79013906	12.469%
1080	14.51%	0.69671846	12.744%

Exercise Rate (X) =	11.50%	Swap to pay and receive LIBOR
3 Year Swap R =	12.744%	Swap to pay LIBOR and receive
The Net Effect =	1.2444%	
Notional Amount	10,000,000	
Payments	$ 124,445	per year
PV Payments	$ 296,144	This what the Swaption is worth at Expiration

4. Forward Swaps

The entire semester we cover Options and Forward contracts. Because these are options on Swaps, there will also be forward contracts on Swaps – called Forward Swaps. This is a contract of two parties to enter into a swap at a specific fixed rate – one to give and one to receive. Since there is forward contract there is no cash up front.

MPK Example

				FORWARD SWAP - YEARS					
Term (Days)	**Rate (given)**	**Discount Bond Price**		**Years**	**Forwared Rate**	**0**	**1**	**2**	**3**
360	9.00%	0.917431		360	9.00%				
720	10.06%	0.832501		720	10.06%		0.108	0.1161	0.1238
1080	11.03%	0.751371		1080	11.03%				
1440	12.00%	0.675676		1440	12.00%				
1800	12.95%	0.60698		1800	12.95%				
					PV =		0.9025	0.8116	0.7291
					Sum	2.4433			
			The Rate of Forward Swap =			0.1109			