LECTURE 2
MANAGING BOND PORTFOLIOS

(Chapter 16)

Interest Rate Risk /Sensitivity – Calculating Duration and Convexity

$$D_{Mac} = \frac{\sum_{t=1}^{N} \frac{CF_t}{(1+i)^t} t}{V_B}$$

$$C = \frac{\frac{1}{(1+i)^2}\left[\sum_{t=1}^{N} \frac{CF_t}{(1+i)^t}\left(t^2+t\right)\right]}{V_B}$$

Effective Duration (**expressed in percentage %)** is a measure of the sensitivity of the asset's price to interest rate movements. It broadly corresponds to the length of time before the asset is due to be repaid. This **duration** is equal to the ratio of the percentage reduction in the bond's price to the percentage increase in the redemption yield of the bond (or vice versa) (Lamda). For example, a 15 year bond with duration of 7 would fall approximately 7% in value if interest rates increase by 1.0%.

Linear Calculation - Duration is the approximate percentage change in price for a 100 basis point change in rates. To compute duration, you can apply the following equation that was presented earlier in the guide.

Price if yield decline - price if yield rise / 2(initial price)(change in yield in decimal)

Let's make:
Δy = change in yield in decimal (Δ = "delta")
V1 = initial price

V2 = price if yields decline by Δy
V3 = price if yields increase by Δy

Duration = V2 - V3 / 2(V1)(? y)

Example:
Stone & Co 9% of 10 are option free and selling at 106 to yield 8.5%. Let's change rates by 50 bps. The new price for the increase in 50 bps would be 104 and the new price for a decrease in rates would be 109. Then:

Answer:
Duration = 109 - 104 / 2 *(106) * (.005)
Duration = 5 / 1.06
Duration = 4.717

This means that for a 100 basis point change, the approximate change would be 4.717%

Price Change Given the Effective Duration and Change in Yield
Once you have computed the effective duration of a bond it is easy to find the approximate price change given at change in yield.

Modified Duration = 1/(1+yield/k)[1 x pvcf1 + 2 x pvcf2 +...+n x pvcfn / k x Price

Approximate Percent Price change = - duration x change in yield x 100

<u>Example:</u>
Using the duration for 4.717% obtained from the previous example, let's see the approximate change for a small movement in rates such as a 20 bps increase.

Percentage Price Change = - 4.717 x (+0.0020) x 100 = -.943%
And for a large change, a 250 bps increase:

Percentage Price Change = -4.717. (+0.0250) x 100 = -11.79%

As noted before, these changes are estimates. For small changes in rates, the estimate will be almost dead on. For larger movements in rates, the estimate will be close but will nderestimate the new price of the bond regardless of whether the movement in rates is up or down. Later we will discuss convexity which is the derivative to duration – it accounts for the inaccuracies in the calculation of a linear duration line.

<u>Macaulay Duration **(expressed in years)**</u> is the PV-weighted time to receive each cash flow, defined as:

Weighted Average $Wt = [cf / (1 + y)^t] / \text{Bond Price}$

Y = yield to maturity
T=time

$D = \sum t \times Wt$

MACAULAY DURATION

100	K	L	M	N	O	P	Q
101	**Duration**						
102							
103	Int.Rate =	10%					P x 0
104			Time until	Payment	PV of Pmt	%	Duration
105			Payments		DR = 10%	Weight	
106	8% coupon bond		**1**	**80**	**72.727**	7.65%	0.0765
107			**2**	80	**66.116**	6.96%	0.1392
108			**3**	1080	**811.420**	85.39%	2.5617
109					950.263	100.00%	2.7774
110							
111		Duration					
112							
113	Zero Bond	will be 3 years					
114							
115							
116							

Duration is a key concept in bond portfolio management for at least 3 reasons:

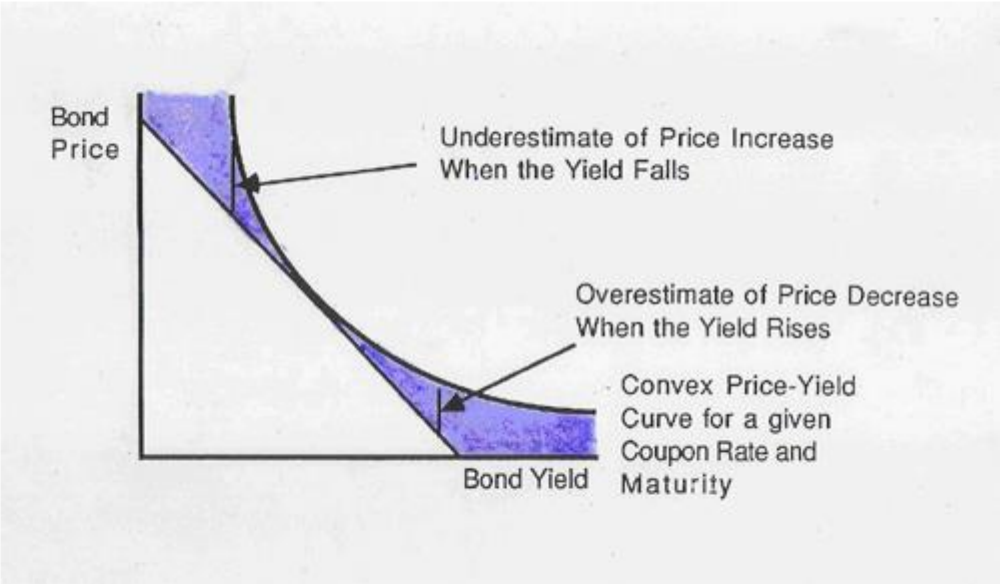
1. It's a simple summary measure of the effective average maturity of the portfolio
2. It turns out to be an essential tool in immunizing portfolios from interest rate risk.
3. Duration is the measurement of the interest rate sensitivity of a bond portfolio.

Convexity
Convexity is a further measure, or second derivative measure, of the sensitivity of the duration of a bond to changes in interest rates. There is an inverse relationship between convexity and sensitivity - in general, the higher the convexity, the less sensitive the bond price is to interest rate shifts, the lower the convexity, the more sensitive it is.

Duration is a linear measure or 1st derivative of how the price of a bond changes in response to interest rate changes. As interest rates change, the price is not likely to change linearly, but instead it would change over some curved function of interest rates. The more curved the price function of the bond is, the more inaccurate duration is as a measure of the interest rate sensitivity.

Like duration, convexity is also useful for comparing bonds or bond portfolios. If two bonds offer the same duration and yield but one exhibits different convexity, changes in interest rates will affect each bond differently.

Unfortunately, duration has limitations when used as a measure of interest rate sensitivity. The statistic calculates a linear relationship between price and yield changes in bonds. In reality, the relationship between the changes in price and yield is convex. In Figure 1, the curved line represents the change in prices given a change in yields. The straight line, tangent to the curve, represents the estimated change in price via the duration statistic. The shaded area shows the difference between the duration estimate and the actual price movement. As indicated, the larger the change in interest rates, the larger the error in estimating the price change of the bond.



Convexity, which is a measure of the curvature of the changes in the price of a bond in relation to changes in interest rates, is used to address this error. Basically, it measures the change in duration as interest rates change. The formula is as follows:

$$C = \frac{1}{B}\frac{d^2\left(B(r)\right)}{dr^2}$$

C is convexity, B is the bond price, r is the interest and d is duration

Annual Analysis

MACAULAY DURATION AND CONVEXITY - Annual Analysis

145	K	L	M	N	O	P	Q
146	**Period**	**Cash Flow**	**PV Cash Flow**	**Weighted**	**Duration Calc**	**Factor years**	**Convexity Calc**
147	1	$80.00	72.73	8.3%	0.08292	2	145.45
148	2	$80.00	66.12	7.5%	0.15076	6	396.69
149	3	$80.00	60.11	6.9%	0.20558	12	721.26
150	4	$80.00	54.64	6.2%	0.24919	20	1,092.82
151	5	$80.00	49.67	5.7%	0.28317	30	1,490.21
152	6	$80.00	45.16	5.1%	0.30891	42	1,896.63
153	7	$80.00	41.05	4.7%	0.32763	56	2,298.95
154	8	$80.00	37.32	4.3%	0.34040	72	2,687.08
155	9	$80.00	33.93	3.9%	0.34813	90	3,053.50
156	10	$1,080.00	416.39	47.5%	4.74727	110	45,802.54
157			**877.11**	100.0%	**7.04395**		**56.14**
			PRICE		**DURATION**		**CONVEXITY**

Interest rates are constantly changing and add a level of uncertainty to fixed-income investing. Duration and convexity allow investors to quantify this uncertainty and are useful tools in the management of fixed-income portfolios.

Semi Annual Analysis

100	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
101	**MACAULAY DURATION AND CONVEXITY - Semi annual analysis**													
102	**Sensitivity to interest rate movements**							IRR=	10.0000%					
103				*=-PV(M108/M109,M107*M109,M106*M105/M109,M105)*										
104	Bond Price		$875.38			**If Yield Changes By**		**1.00%**						
105	Face Value		**1,000**			Bond Price Will Change By		-54.63	-6.24%	=+R105/M104				
106	Coupon Rate		**8.00%**						=-(M104+PV((M108+R104)/M109,M107*M109,M106*M105/M109,M105))					
107	Life in Years		**10**			Modified Duration Predicts		-57.03	=(-M112*R104*M104)					
108	Yield		**10.00%**			Convexity Adjustment		2.47	*=0.5*M113*R104^2*M104*					
109	Frequency		**2**			Total Predicted Change		-54.56	*=+R107+R108*					
110														
111	**Macaulay Duration**		**6.84**	*=+O138*		Actual New Price		$820.74	*=-PV((M108+Q104)/M109,M107*M109,M106*M105/M109,M105)*					
112	Modified Duration		6.51	*=+M111/(1+M108/M109)*		Predicted New Price		$820.82	*=+M104+R109*					
113	Convexity		56.49	*=+S137/M104/M109^M109*		Difference		$0.08	*=+R112-R111*					
114														
115	**Period**	**Cash Flow**	**PV Cash Flow**	**Weighted**	**Duration Calc**	**PV CF / ((1+i/2)^2) (semi-annual adjustment)**	**Factor years**	**Convexity Calc**						
116	0	($875.38)												
117	1	40.00	38.10	4.352%	0.04352	34.554	2.000	69.11						
118	2	40.00	36.28	4.145%	0.08289	32.908	6.000	197.45						
119	3	40.00	34.55	3.947%	0.11842	31.341	12.000	376.09						
120	4	40.00	32.91	3.759%	0.15037	29.849	20.000	596.97						
121	5	40.00	31.34	3.580%	0.17901	28.427	30.000	852.82						
122	6	40.00	29.85	3.410%	0.20459	27.074	42.000	1,137.09						
123	7	40.00	28.43	3.247%	0.22732	25.784	56.000	1,443.92						
124	8	40.00	27.07	3.093%	0.24742	24.557	72.000	1,768.07						
125	9	40.00	25.78	2.946%	0.26510	23.387	90.000	2,104.85						
126	10	40.00	24.56	2.805%	0.28052	22.273	110.000	2,450.08						
127	11	40.00	23.39	2.672%	0.29388	21.213	132.000	2,800.10						
128	12	40.00	22.27	2.544%	0.30533	20.203	156.000	3,151.62						
129	13	40.00	21.21	2.423%	0.31503	19.241	182.000	3,501.80						
130	14	40.00	20.20	2.308%	0.32310	18.324	210.000	3,848.14						
131	15	40.00	19.24	2.198%	0.32970	17.452	240.000	4,188.45						
132	16	40.00	18.32	2.093%	0.33493	16.621	272.000	4,520.86						
133	17	40.00	17.45	1.994%	0.33892	15.829	306.000	4,843.78						
134	18	40.00	16.62	1.899%	0.34177	15.076	342.000	5,155.85						
135	19	40.00	15.83	1.808%	0.34357	14.358	380.000	5,455.92						
136	20	1,040.00	391.97	44.777%	8.95533	355.524	420.000	149,320.02						
137		**Total**	**875.38**	100%	13.68074	793.99356		197,783.01						
138		PRICE		DURATION	**6.84037**		CONVEXITY	**56.49**						

Bond Terminology

Accrued Interest

Accrued interest is the interest that has been earned, but not yet been paid by the bond issuer, since the last coupon payment. Note that interest accrues equally on every day during the period. That is, it does not compound. So, halfway through the period, you will have accrued exactly one-half of the period's interest payment. It works the same way for any other fraction of a payment period.

Banker's Year

A banker's year is 12 months, each of which contains 30 days. Therefore, there are 360 (not 365) days in a banker's year. This is a convention that goes back to the days when "calculator" and "computer" were job descriptions instead of electronic devices. Using 360 days for a year made calculations easier to do. This convention is still used today in some calculations such as the Bank Discount Rate that is used for discount (money market) securities.

Bond

A bond is a debt instrument, usually tradable, that represents a debt owed by the issuer to the owner of the bond. Most commonly, bonds are promises to pay a fixed rate of interest for a number of years, and then to repay the principal on the maturity date. In the U.S. bonds typically pay interest every six months (semi-annually), though other payment frequencies are possible. Bonds are issued by corporations, banks, state and local governments (municipal bonds), and the federal government (Treasury Notes and Bonds).

Call Date

Some bonds have a provision in the indenture that allows for early, forced, redemption of the bond, often at a premium to its face value. Bonds that have such a feature usually have a series of such dates (typically once per year) at which they can be called. This series of dates is referred to as the call schedule.

Call Premium

The extra amount that is paid by a bond issuer if the bond is called before the maturity date. This is a sweetener that is used to make callable bonds attractive to investors, who would otherwise prefer to own non-callable bonds.

Clean Price

The "clean price" is the price of the bond excluding the accrued interest. This is also known as the quoted price.

Coupon Payment

The is the actual dollar amount that is paid by the issuer to the bondholders at each coupon date. It is calculated by multiplying the coupon rate by the face value of the bond and then dividing by the number of payments per year.

Coupon Payment Date

The specified dates (typically two per year) on which interest payments are made.

Coupon Rate

The stated rate of interest on the bond. This is the annual interest rate that will be paid by the issuer to the owners of the bonds. This rate is typically fixed for the life of the bond, though variable rate bonds do exist. The term is derived from the fact that, in times past, bond certificates had coupons attached. The coupons were redeemed for cash payments.

Current Yield

A measure of the income provided by the bond. The current yield is simply the annual interest payment divided by the current market price of the bond. The current yield ignores the potential for capital gains or losses and is therefore not a complete measure of the bond's rate of return.

Day-count Basis

A method of counting the number of days between two dates. There are several methods, each of which makes different assumptions about how to count. **30/360** (a banker's year) assumes that each month has 30 days and that there are 360 days in a year. **Actual/360** counts the actual number of days, but assumes that there are 360 days in a year. **Actual/Actual** counts the actual number of days in each month, and the actual number of days in a year. In Excel bond functions, 0 signifies 30/360, 1 specifies actual/actual, 2 is actual/360, 3 is actual/365 (which ignores leap days), and 4 represents the European 30/360 methodology.

Dirty Price

The "dirty price" is the total price of the bond, including accrued interest. This is the amount that you would actually pay (or receive) if you purchase (or sell) the bond.

Face Value

The principal of a bond is the notional amount of the loan. It is also called the **principal** or **par** value of the bond, and represents the amount that will be repaid when the bond matures.

Indenture

The legal contract between a bond issuer and the bondholders. The indenture covers such things as the original term to maturity, the interest rate, interest payment dates, protective covenants, collateral pledged (if any), and so on.

Maturity Date

The date on which the bond ceases to earn interest. On this date, the last interest payment will be made, and the face value of the bond will be repaid. This is also sometimes known as the **redemption date**.

Redemption Value

This is typically the same as the face value of a bond. However, for a callable bond, it is the face value plus the call premium. In other words, this is the entire amount that will be received when the bond is redeemed by the issuer.

Settlement Date

The date on which ownership of a security actually changes hands. Typically, this is several days after the trade date. In the US markets, the settlement date is usually 3 trading days after the trade date (this is known as T+3). For bonds, a purchaser begins to accrue interest on the settlement date.

Term to Maturity

The amount of time until the bond stops paying interest and the principal is repaid.

Yield to Call

Same as yield to maturity, except that we assume that the bond will be called at the next call date. Also known as yield to first call. Frequently, the yield to all call dates is calculated, and then we can find the worst-case, which is known as the yield to worst.

Yield to Maturity

The yield to maturity (YTM) of a bond is the compound average annual expected rate of return if the bond is purchased at its current market price and held to maturity. Implicit in the calculation of the YTM is the assumption that the interest payments are reinvested for the life of the bond at the same yield. The YTM is the internal rate of return (IRR) of the bond.

Yield to Worst

The lowest of all possible yields for the bond. It is calculated by determining the minimum of the yield to maturity or any of the various yields to call date.