Chapter 4
Sharpe Ratio, CAPM, Jensen's Alpha, Treynor Measure & M Squared

This chapter will continue to emphasize the risk and return relationship. In the previous chapters, the risk and return characteristics of a given portfolio were measured at first versus other asset classes and then second, measured to market benchmarks. This chapter will re-emphasize these comparisons by introducing other ways to compare via performance measurements ratios such the Share Ratio, Jensen's Alpha, M Squared, Treynor Measure and other ratios that are used extensively on wall street.

Learning Objectives

After reading this chapter, students will be able to**:**

- Calculate various methods for evaluating investment performance
- Determine which performance ratios measure is appropriate in a variety of investment situations
- Apply various analytical tools to set up portfolio strategy and measure expectation.
- Understand to differentiate the between the dependent and independent variables in a linear regression to set return expectation
- Determine how to allocate various assets classes within the portfolio to achieve portfolio optimization.

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AUTHOR'S NOTES:
In the spring of 2006, just 2 years before the worse financial crisis the U.S. has ever faced since the 1930's, I visited few European countries to promote a new investment opportunity for these managers who only invested in stocks, bonds and real estate. The new investment opportunity, already established in the U.S., was to invest equity in various U.S. Collateral Loan Obligations (CLO). The most difficult task for me is to convince these managers to accept an average 10-12% return when their portfolio consists of stocks, bonds and real estate holdings enjoyed returns in excess of 30% per year for the last 3-4 years. I was told that it was too much work for them to include a new opportunity that only gives returns 3.0 times lower than their existing portfolio. My marketing pitch of course was that CLOs had an unlevered return of approximately 6.0% with standard deviation, measuring volatility, of 2.0%. The relationship between return and risk was among the best when comparing to their holdings. I was also showing very low correlations with other assets which of course can help any portfolio to achieve higher efficiency. I am not sure if it

was just timing of my marketing pitch, or behavioral or how the managers got paid their bonuses - driven by total return, but I failed to convince them to invest in my product. I failed to convince them to diversify a portion of their portfolio into a lower volatility asset class in case the markets turn south. Evaluating performance of the portfolio based on average return alone is not very useful. These returns are needed to be adjusted for risk. This relationship between return and volatility, explained in pervious chapters, are the basis for introducing other ratios, such as the Sharpe Ratio or the Jensen's Alpha.

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KEY TAKEAWAYS:

- *A portfolio analyst needs to value the portfolio based on trends, market benchmarks and expectation.*
- *Evaluating performance of the portfolio based on an average return alone is not that useful The analyst needs to adjust such returns to risk before it could be meaningfully compared.*
- *Many portfolio value measurements ratios were set-up to give you a comprehensive understanding of the performance as follows:*
    - *The Sharpe ratio measures the excess return to standard deviation.*
    - *The Treynor ratio measures the excess return beta*
    - *The Jensen's Alpha is the excess return over the market index*
    - *The M square focuses on total volatility as a measured of adjusted risk compared to the market benchmark*

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Setting-up the basis for portfolio measurement: re-introduction to basic return characteristics

Repeating of what was discussed in Chapter 1, it is important to mention the conventional theories of measuring performance returns as the basis for introducing other ratios.

Average Rate of Returns:

The first ratio that was introduced was the Holding Period Return (HPR). **In its simplest form is the best approach to calculate the ongoing return of any asset class or portfolio. The numerator represents all the cash activity of an investment including the trading cash inflows and outflows, any dividends received minus any taxes paid and any hedging costs, etc. over the denominator representing the initial investment as follows:**

$$\text{HPR} = \frac{CF}{I}$$

Where **CF** is the Cash Flow (inflow and outflow) during the investment period and **I** is the initial investment. For example, if an investor buys the stock for \$100 and sells it for \$120 and during the investment he or she received \$2 dividend then the cash flow on the numerator will be \$120 of proceeds for selling the stock plus \$2 of cash dividend received (cash inflow) minus the initial investment of \$100 (cash outflow) the net cash flow will be \$22 (\$120 + \$2 - \$100). The HPR will be calculated by dividing the net cash flow of \$22 by the initial investment of \$100 resulting to a 22% return:

$$\frac{(120 - 100 + 2)}{100} = \frac{22}{100} = 0.22 = 22\%$$

For a quick analysis of expected return, the HPR ratio which represents the relationship between cash flow to the initial investment can be found in many applications on various asset classes as described in chapter 1.

The next average return that was introduced in Chapter 1 is the annual return since the portfolio managers have access to such returns when comparing other investments or market benchmark as well as trend analysis. Also, since a lot of portfolios could be levered to include margin loans of debt funds, which are structured with set annual costs, it's easier to compare the portfolios on an annual basis, hence using the Internal Rate of Return (IRR). **This IRR method, adds all the annual HPR's that accounts for each annual activity found on the numerator of this ratio to different annual investment activities as the denominators and is weighted by year to represent an average annual return** in figure 4.1 below:

Insert Figure 4.1

Internal Rate of Return (IRR)

Dollar Weighted Return (Even Annual Payments)

	0	1	2	3	4
Net CF (\$)	-100	5	5	5	105

	Excel	B	C	D	E	F
10	IRR	0	1	2	3	4
11	5.00%	-100	5	5	5	105

=IRR(B11:F11)

Dollar Weighted Return (Uneven Annual Payments

	0	1	2	3	4
Net CF (\$)	-100	-9	-5	26	110

1 = + (-0.1 / (1+IRR) + (-0.5 / (1+ IRR) ^2 + (0.8 / (1+ IRR)^3) + (1.0 / (1+IRR)^4

	Excel	B	C	D	E	F
10	IRR	0	1	2	3	4
11	4.96%	-100	-9	-5	26	110

=IRR(B11:F11)

Figure 4.1

Just comparing two assets classes to each other or to the bench mark using average return does not tell the whole story how these assets classes performed. To correspond a more effective comparison, they needed to be adjusted for risk or volatility. Also, these returns need also to be calculated after subtracting the risk-free rate. Comparing absolute returns between one year to another could not correctly reflect a true comparison as the risk-free rate that changes from one year to another. The difference between the absolute return and risk-free rate is called the risk premium return which is the basis for many portfolio performance ratios discussed later in this chapter.

Setting-up the basis for portfolio measurement: re-introduction to basic volatility characteristics

Again, repeating of what was discussed in Chapter 1, it is important to re-introduce the conventional theories of measuring performance returns and as it relates to its volatility - a basic measurement of risk. Two portfolios with the same average return do not tell whole story. Take the following two stocks shown in figure 4.2 sorted by minimum to maximum values. Both stocks averaged at $40 but Stock A had a largest variance of prices ranging from $20 to $60 as compared to stock B that the prices stayed between $30 to $50

Insert Figure 4.2

Comparing two stocks

Same average return with different variance

	Stock A $ price	Stock B $ price
	$ 20.00	$ 30.00
	$ 30.00	$ 35.00
Range	$ 40.00	$ 40.00
	$ 50.00	$ 45.00
	$ 60.00	$ 50.00
Average	$ 40.00	$ 40.00
$ Variance (max-min)	$40.00	$20.00
Avg/$Variance	1.00x	2.00x

Figure 4.2

From Portfolio Efficiency to Optimization

EFFICIENT FRONTIER → $\frac{\Delta r}{\Delta \sigma} > 1$ → OPTIMIZATION

Chapter 2 covered the process by way the portfolio manager finds the point of efficiency, or efficient frontier. This represents the highest the highest possible return with the lowest possible risk. Once that point is determined, the portfolio manager's next task to go from efficiency to optimization. The optimization can be achieved by moving from the efficient frontier to seek additional return at minimum rate of change of risk. From the point of efficiency, the portfolio manager is seeking to achieve even a higher return delta (rate of change) but as we discussed it will also come with higher risk. The optimization point is where additional return, or the rate of change going form bonds to stocks should be lower than the rate of change of the additional risk – basically, higher return delta at lower risk delta. That achievement is called Optimum point and the basis of the Sharpe Ratio. Figure 4.3 below shows a basic illustration of the optimization process.

Insert Figure 4.3

FROM EFFICIENCY TO OPTIMIZATION

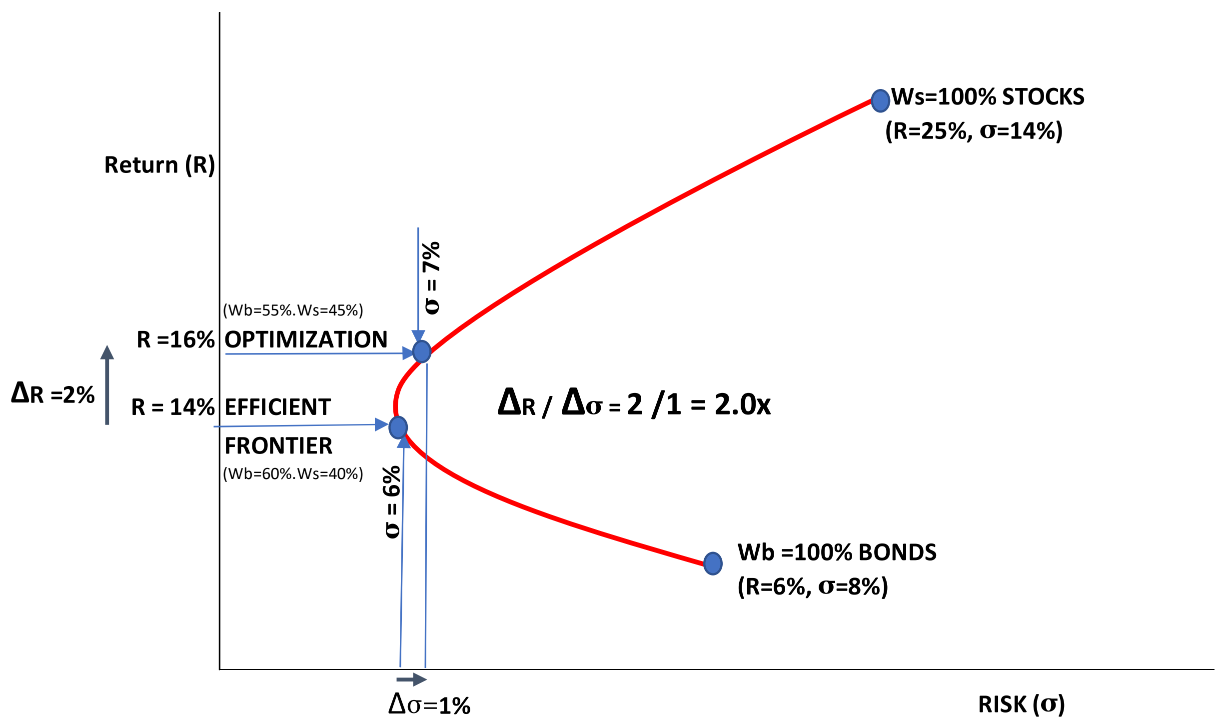

Fig. 4.3

Figure 4.3 above shows that the efficient frontier is achieved at 14% return (R) and 6.0% Risk (σ) as the analyst trades out of 100% bonds that has risk and return of 6% and 8%, respectively to buy stocks. As it trades out of bonds to stocks, the return is increasing, and the risk is decreasing to the point that the slope of the graph shift towards higher risk. After achieving that efficient frontier – assuming having 60% bonds and 40% stocks, the analyst wants to continue to buy stock by selling bonds seeking higher returns. The optimization point is where the rate of change (ΔR) of the return is higher than the rate of change ($\Delta\sigma$) of the risk. In this case a 2.0% higher rate of return is achieved by only giving up 1.0% of standard deviation calculating ΔR / $\Delta\sigma$ = 2 /1 = 2.0x assuming at 55% of Bonds and 45% of Stocks.

The following ratios are the most common ratios used to measuring the performance of a given portfolio. It's important to note, emphasized by this author in chapter 1, that the best way of assessing or analyzing anything, in this case the portfolio, is for the analyst to keep answering the following three questions:

1. **How well the portfolio is performing versus last year or the years before**
2. **How well the portfolio is performing versus other portfolios or the market benchmark**
3. **How well the portfolio is performing versus expectation given various scenarios**

Sharpe Ratio

The Sharpe Ratio was named after William Sharpe who devoted all his adult life analyzing the relationship between risk and return among various asset classes and as compared to their respective markets. **The Sharpe ratio, in its basic form is the relationship between return (on the numerator) to the risk (denominator)**. The numerator though is adjusted to reflect the risk premium which is calculated by taking the absolute rate of return and subtracting the risk-free rate which, by definition, has little to no risk, for a given time. The denominator representing the risk that is measured by the volatility of the investment return or the standard deviation of the asset class for the same time period. The Sharpe Ratio formula is as follows:

$$SR = \frac{R_P - Rf}{\sigma_P}$$

Where R_P is the absolute return of a given portfolio, Rf is the risk-free rate and σ_P is the standard deviation of the portfolio.

The resulting number from this Sharpe Ratio reflects the relationship between numerator and denominator. For example, a Sharpe Ratio of 0.5 shows that every 1.0% of return is a corresponding 2.0% of risk. A Sharpe Ratio of 2.0 shows that every 2.0% of return has a corresponding risk of only 1.0%. The higher the ratio the higher the comfort level the analyst gets when seeking additional returns. Of course, the result, though it gives you that relationship, it's more important to apply the authors "3-questions" assessment rule, mentioned above, to determine a better framework for the analysis including how well this ratio versus years in the past, how well this ratio is measured versus other portfolios and has this ratio beat expectation.

The illustration expressed previously in figure 4.3 shows that the optimization point is achieved when the portfolio is broken down to 55% bonds and 45% stocks achieving 16% return with a 7% standard deviation. Assuming a 3.0% risk free rate that has 0% standard deviation the Sharpe Ratio is calculated as follows:

$$at\ the\ optimazation\ point,\ SR = \frac{R_P - Rf}{\sigma_P} = \frac{16-3}{7} = 1.86$$

Showing that every 1.86% increase in return it has an equivalent 1.0% of risk. The optimization point is the point with the highest possible Sharpe Ratio number. Using the same illustration (figure 4.3) to calculate the Sharpe Ratio using the efficient frontier return and risk numbers the ratio calculates as follows:

$$at\ the\ efficiency\ point,\ SR = \frac{R_P - Rf}{\sigma_P} = \frac{14-3}{6} = 1.83$$

Showing that every 1.83% increase in return it has an equivalent 1.0% of risk. Even though the efficient frontier which represents the highest possible return with the lowest possible risk is not longer the case when introducing the risk-free rate. After the risk premium is calculated the optimization point has a higher Sharpe Ratio that the efficient frontier.

Capital Asset Pricing Model (CAPM)

Another concept that was extensively developed by William Sharpe in the early 1960's is the Capital Asset Pricing Model (CAPM). CAPM is a formula that was developed to calculate the expected return of any risky asset class (ER_i) as compared to the systematic market risk. This formula that will be used extensively in later chapters not only for portfolio management applications but also to be used as a discount rate for determining a present value of the equity invested in a firm. The formula is as follows:

$$ER_i = R_f + \beta(ER_m - R_f)$$

Where R_f is the risk-free rate, β is the beta, and ER_m is the expected market return.

If the investor's expectation is risk-free then the formula will be $ER_i = R_f$. If the investor anticipates taking additional risk, then the second part of the equation $\beta(ER_m - R_f)$ represents the risk premium return over above the risk-free rate. This risk premium is adjusted by the beta (β) which represents the multiple of risk as compared to the market. As described in earlier chapters, if the beta (β) equals 1 then the investor expects the set market premium return. If a certain stock is trading at beta (β) of 2.0 then the expected premium return will be adjusted to be twice as much as the market premium return expectation. If the beta (β) is less than 1 then that specific stock premium return is anticipated to be less than the marker premium return.

The objective of CAPM is set as the basis for evaluating if the stock is fairly valued as compared to the market. For example, let's assume that the investor is looking to buy XYZ Inc.'s stock that has a beta (β) of 1.5x, which means that the volatility of such stock is 1.5x the volatility of the total equity market index. If the market is anticipating growing 10.0% this year, then the return of such investment should grow at 15.0% (1.5 x 10%). The CAPM formula though adjusts for risk-free rate after establishing the market risk premium return $(ER_m - R_f)$ or (10% -R_f) so the expected risk premium return for such stock is 1.5x the market premium. Assuming the risk-free rate is 2.0%, then the expected investment return is calculated at 14% as follows:

$$ER_i = R_f + \beta(ER_m - R_f) = 2\% + 1.5\ (10\% - 2\%) = 2\% + 12\% = 14\%$$

CAPM is used as the basis of the minimum expectation of an investor that is seeking when evaluating a portfolio of stocks or a single stock adjusted to market fluctuations. Later we will discuss that, when valuating investments in specific companies, the CAPM is used a first guideline for the minimum required return before deciding to invest. Any additional return over the calculated CAPM expectation is considered that the investor exceeded that minimum return and that will suggest to go ahead with the investment. In portfolio theory, this excess called Jensen's Alpha is in the next section.

Jensen's Alpha Ratio

The Jensen's Alpha developed by mutual fund manager Michael Jensen in the late 1960's **is a formula that determines the average return over or below (negative alpha) the expectation calculated by the CAPM as described in the previous section.** As mentioned previously, CAPM represents the minimum expected return of a portfolio or a single stock adjusted to the market expectation. Jensen's alpha, or simply alpha (α), if positive, represents the excess return over CAPM. The formula is as follows:

$$\alpha = R_i - [R_f + \beta(R_m - R_f)] \ \ or \ \ \alpha = R_i - CAPM$$

Where R_i is the realized return on the investment of the portfolio, R_f is the risk-free rate, β is beta and R_m is the market index return.

To analyze the performance of an asset manager as compared to other asset managers, the analyst must not only look at the actual portfolio returns but the Alpha (α) that managers are generating for their respective portfolio. For example, if two portfolios of investments, such as a mutual funds, generate 20% return, the next question the analyst will ask these asset managers what their risk-adjusted return is against the market and whether that return has exceeded such market adjusted return. Jensen's Alpha is one number that could be used to compare these two portfolios. First, if the value is positive demonstrating that they beat the market and then what is the actual alpha (α) when comparing these two portfolios. Using the 20% example, let's assume portfolio Z had beta (βz) of 1.3 and portfolio X had beta (βx) of 1.5. Let's assume the overall market returned 10% and risk-free rate is 2.0% for that period. The following formulas calculates the alphas for portfolio Z (αz) and portfolio X (αz):

$$\alpha z = R_i - [R_f + \beta(R_m - R_f)] = 20\% - [2\% + 1.3\ (10\% - 2\%) = 7.6\%$$

$$\alpha z = 7.6\%$$

$$\alpha x = R_i - [R_f + \beta(R_m - R_f)] = 20\% - [2\% + 1.5\ (10\% - 2\%) = 6.0\%$$

$$\alpha x = 6.0\%$$

The alpha for portfolio Z at 7.6% is higher than portfolio X 0f 6.0% demonstrating that despite both have beaten the market showing positive alpha (α), portfolio Z had a better performance when adjusting for risk which of course is determine by the lower beta (β). Let's use another example where portfolio Z shows 19% return and portfolio X shows 20%. Just looking at the total returns it's obvious at first to conclude that portfolio X has slightly outperformed portfolio Z (20% versus 19%). When calculating the Alphas though we can see that portfolio Z has a better performance metrics despite the lower overall return after adjusting for risk, as follows:

$$\alpha z = R_i - [R_f + \beta(R_m - R_f)] = 19\% - [2\% + 1.3\ (10\% - 2\%) = 6.6\%$$

$$\alpha z = 6.6\%$$

$$\alpha x = R_i - [R_f + \beta(R_m - R_f)] = 20\% - [2\% + 1.5\ (10\% - 2\%) = 6.0\%$$

$$\alpha x = 6.0\%$$

The cliché "seeking alpha" is a statement that most asset managers like to say when pitching their investment thesis and one number that they represent historically to show their ability the met such expectation.

Treynor Ratio

Both the Sharpe Ratio and Treynor Ratio measures the relationship between risk and return. Both ratios have the same numerator but different denominators. The numerator representing premium return ($R_P - Rf$) and the denominators, though it addresses the risk, it has slightly different objective. The Sharpe Ratio focuses on the relationship between the portfolio risk premium return and standard deviation of the portfolio that is expresses as deviation percentage (0-1) from the portfolio mean (average). **The Treynor Ratio focuses on the relationship between the portfolio risk premium return and beta (β) of the portfolio that is expressed in factors (positive or negative) or multiple of the market premium risk.** The formula is as follows:

$$T = \frac{R_P - Rf}{\beta_P}$$

Jack Treynor, an American economist, who is credited and was awarded for many finance and portfolio management concepts in his career presented this specific ratio with the objective to measure the performance ratio that could apply across all investors regardless of their personal risk appetite. In many of his papers he made many suggestions that they were really two types of volatility risks: the stock market risk and the individual risk of the portfolio or specific stock – measured by beta (β). The beta coefficient, discussed in previous chapters, is simply the volatility or the line's slope (steepness) of a portfolio to the market itself. The higher the line's slope or steepness the better the risk-return tradeoff.

The Treynor ratio, also known as the reward-to-volatility ratio, is designed to assess the portfolio's performance against the benchmark. Instead of measuring a portfolio return only against the risk-free rate, the ratio examines how well a portfolio outperforms the market. Using the previous example, let's assume the market benchmark had a 10% return which represents beta (β=1), portfolio Z and portfolio X returned 19% and 20% respectively. Portfolios Z and X had betas (β) of 1.30 and 1.50, respectively. Also, let's assume the risk-free rate (treasury bills) is 2.0%. The Treynor value of each is calculated as follows:

Market $\quad Tm = \frac{R_m - Rf}{\beta_m} = \frac{.10 - .02}{1} = .080 = 8.000\%$

Portfolio Z $\quad Tz = \frac{R_z - Rf}{\beta_z} = \frac{.19 \quad .02}{1.3} = .1307 = 13.077\%$

Portfolio X $\quad Tx = \frac{R_x - Rf}{\beta_x} = \frac{.20 - .02}{1.5} = .120 = 12.000\%$

The higher the Treynor ratio (T), the more efficient the portfolio. Like the Sharpe Ratio discussed above, if the analyst was only evaluating the portfolio on return performance alone, he or she may have recognized that portfolio X have returned the best results. The Treynor ratio is adjusted to reflect the risk adjusted to the market.

M Squared Ratio

Like the Treynor ratio, M squared (M^2) is another ratio that measures the risk adjusted return of the portfolio relative to the market benchmark. **M^2 measures the difference between the excess**

return of the portfolio over the market where the portfolio has the same portfolio as the market. Unlike the Sharpe ratio that is measured in units of return versus risk, M^2 is expressed in percentage return which is easier for the investor to read when analyzing a portfolio. M^2 is one of the newest modern portfolio measurement methods only developed in 1997 by the Nobel prize winner Franco Modigliani and his granddaughter, Leah Modigliani, hence the concept of M squared. The formula is as follows:

$$M^2 = SR \cdot \sigma_m + R_f$$

Where SR is the Sharpe ratio of the risky portfolio, σ_m is the market benchmark standard deviation and R_f is the risk- free rate. The M^2 ratio can be written also as follows:

$$M^2 = \frac{R_p - R_f}{\sigma_p} \cdot \sigma_m + R_f$$

Where is R_p is the portfolio return, σ_p is the portfolio's standard deviation, σ_m is the market benchmark standard deviation and R_f is the risk- free rate. Rearranging the formula,

$$M^2 = RPRp \,.\frac{\sigma_m}{\sigma_P} \; + R_f$$

Where is RPR_p is the portfolio risk premium return, σ_p is the portfolio's standard deviation, σ_m is the market benchmark standard deviation and R_f is the risk- free rate.

Let's use the same information used above to compare portfolio Z to portfolio X to measure their M^2s. Assuming portfolio Z had returns of 19% with standard deviation of 26% and portfolio X had returns of 20% with standard deviation of 36%. The market bench mark had return of 10% with standard deviation of 18%. The risk-free rate had a return of 2.0%. The M^2s for these portfolios are calculated as follows:

$$M_z^2 = (Rz - Rf)\,.\frac{\sigma_m}{\sigma_z} \; + R_f = (19\% - 2\%)\,\frac{18\%}{26\%} + 2\% = 13.769\%$$

$$M_x^2 = (Rx - Rf)\,.\frac{\sigma_m}{\sigma_x} \; + R_f = (20\% - 2\%)\,\frac{18\%}{36\%} + 2\% = 11.000\%$$

From the calculations above the analyst can conclude that despite the absolute return for portfolio X of 20% is higher than portfolio Z's 19.0%, portfolio Z has a significant higher M^2 of 13.769% versus 11.00%.

Figure 4.4 below compares the measurements portfolio Z to portfolio X and as compared to the stock market benchmark.

Insert Figure 4.4

Portfolio Perfromance Ratio Analysis

	Description	Symbol	Calculation	Stock Portfolio Z	Stock Portfolio X	Stock Benchmark Market (m)
INPUT	Average Return	*R*		19.00%	20.00%	10.00%
	Risk Free Return	*Rf*		2.00%	2.00%	2.00%
	Standard Deviation	*σ*		26.00%	36.00%	18.00%
	Beta	*β*		1.300x	1.500x	1.000x
OUTPUT	Risk Premium Return	*RPR*	*R - Rf*	17.00%	18.00%	8.00%
	Market Premium	*Pm*	*Rm - Rf*	8.00%	8.00%	8.00%
	Capital Asset Pricing Model	*CAPM*	*Rfr + β. Pm*	12.40%	14.00%	10.00%
	Sharpe Ratio	*SR*	*RPR / σ*	0.654	0.500	0.444
	Jensen's Alpha	*α*	*R - CAPM*	6.600%	6.000%	0.000%
	Treynor Measure	*T*	*RPR / β*	13.077%	12.000%	8.000%
	M-Square	M^2	*((σm/σp)*P)+Rf*	13.769%	11.000%	10.000%

Figure 4.4

Other Useful Portfolio Analysis Ratios

There are other useful ratios portfolio managers around the world use to measure their portfolio performance.

Burke Ratio based on Drawdowns instead of Standard Deviation

The Burke ratio also referred to as "a sharper Sharpe ratio" is like the Sharpe Ratio as it also measures the risk-adjusted performance of the portfolio with the same numerator of Rp – Rf but instead of using the portfolio's standard deviation as the denominator, Burke ratio uses the concept of drawdowns. The drawdowns (D) is used primarily by hedge fund managers to calculate how long it takes the investment to recover from a temporary decline its value. It's expressed as a percentage (%) from the previous peak value – measuring the downside risk. The drawdown (D) is the difference between the current value from the peak value divided by the peak value to determine the percentage drop from the highest value D= min [0, (Pt - Pmax)/ Pmax]. The Burk Ratio formula is as follows:

$$BR = \frac{Rp - R_f}{\dot{D}^2}$$

Where Rp is the portfolio return, R_f isrisk-free return and D is the drawdown.

Omega Ratio based on Min/Max Variance instead of Standard Deviation

The omega ratio (Ω) is another risk-adjusted measurement of return. **This ratio is different that the Sharpe ratio where it compares the return to volatility. The Omega ratio uses the higher points of distribution, arguing that these points could show better assessment of volatility as the distribution can be asymmetric with tail risk or negative skewness** – its basically the probability-weighted ratio of gains versus loses for a give return. The formula is a little complex but is written as follows:

$$\Omega = \frac{\int_{ERp}^{\infty}\left(1 + F(x)\right)dx}{\int_{-\infty}^{ERp} F\,(x)dx}$$

Where F(x) is the cumulative probability distribution function of the returns. In other word, the probability that the return is less than x. ERp is the expected or target return threshold selected by the investor which is used as the basis to compare all the points of gains versus all the points of loses. A higher result shows that the portfolio had more gains than loses comparing to the rate of return threshold represented by ERp. To summarize, the commonly used Sharpe Ratio measures the average of only two points of return distribution to measure volatility based on a normal distribution whereas the Omega Ratio considers all points including the mean, max/min variance, skew and kurtosis. The best way to calculate the Omega Ratio is on excel with a matrix formula. The analyst needs a column of historical returns, a set threshold Return (not the average). The numerator will consist an "=if" statement that takes the sum of all these historical return minus the threshold is its positive divided by the denominator measuring the negative difference between the sum of these investments and the threshold. (Please note that the matrix formula can be run by CNTRL+SHIFT+Return.

Sortino Ratio based on Loses instead of Standard Deviation

The Sortino is a ratio that adjusts for trading loses. Using the standard deviation which is the basis of calculating the Sharpe ratio penalizes both the downside and upside volatility. Sometime investors will like to show the sudden uptick in their portfolio based on decisions they made and using the Sharpe ratio might not be too obvious since it offsets such movement by series of downsides. The Sortino Ratio only penalizes downside risk. The formula is as follows:

$$SoR = \frac{ERp\ - Rf}{\sigma_d}$$

Where, ERp is the expected return of the portfolio, Rf is the risk-free rate and σ_d is the standard deviation of negative asset returns (downside risk). The downside risk is calculating as follows:

$$\sigma_d = \sqrt{\frac{\sum(R_\rho - ER_p)^2 f(t)}{n}}$$

Where, R_ρ are the historical returns of the portfolio, ER_p is Expected Return (threshold), n is the number of years or observations, $f(t)$ represents the arguments that are tested if the returns are higher or lower than the expected return (ER_p) . For example, $f(t)$ = 1 when the total returns are higher than the target return (ER_p) and $f(t)$ = 0 if the historical returns are equal zero or higher than the target return (ER_p).

Portfolio of Stocks and Bonds – Zeus Fund I - Case Study

The previous three chapters focused primarily on the risk and return of the Zeus Fund I portfolio which includes stocks and bonds and how each of this asset class performed against each other and the market. This chapter follow-up on these comparisons and put to test more ratios that the portfolio manager needs to run to better understand the performance of the Zeus Fund I portfolio. Figure 4.5 below shows the Sharpe ratio for the combined portfolio on a total holding period return basis for both levered and unlevered adjusted portfolio. It's worth noting that the standard deviation of the portfolio needs to be adjusted from the leverage. Despite the advantage of leverage where the portfolio return increase from 12.2% to 21.4%, the Sharpe ratio does not change due to leverage. Figure 4.5 below shows a slight change in the Sharpe ratios between the levered and unlevered adjusted portfolio due to moving averages but its negligible. The levered standard deviation is adjusted as follows:

σ (Levered) = σ (Unlevered) x (Rl / Ru)

where, σ is standard deviation, Rl is the Levered Return, R is the Unlevered

σ (Levered) = 9.98% x (12.15% / 21.38%) = 1.16

Insert Figure 4.5

ZEUS Fund I

PORTFOLIO OF STOCKS AND BONDS

CASH FLOWS	MONTHLY IRR	ENTRY 0 June 1 20x1	1 July 1 20x1	2 Aug 1 20x1	3 Sep 1 20x1	4 Oct 1 20x1	5 Nov 1 20x1	6 Dec 1 20x1	EXIT 7 Jan 2 20x2
Beginning Cash		100,000	20,173	23,415	22,396	21,564	21,420	21,932	25,210
Buy/Sell Stock		$ (82,600)	$ -	$ 1,550	$ 2,300	$ -	$ -	$ -	$ 92,600
Buy/Sell Bonds		$ (95,650)	$ 2,875	$ (2,880)	$ (4,075)	$ -	$ 865	$ 3,550	$ 97,000
Stock Dividends			$ 93	$ 90	$ 150	$ 245	$ 63	$ -	$ -
Bond Coupon Received		$ -	$ 875	$ 594	$ 1,344	$ -	$ -	$ -	$ -
Acrued Interest (paid)/Received		$ (1,577)	$ (209)	$ 14	$ (161)	$ -	$ (25)	$ 117	$ 1,927
Loan Principal Increase/Decrease		$ 100,000							$ (100,000)
Loan Interest Payment			$ (417)	$ (417)	$ (417)	$ (417)	$ (417)	$ (417)	$ (417)
Cash Balance Interest Income			$ 25	$ 29	$ 28	$ 27	$ 27	$ 27	$ 27
Cash		$ (20,173)							$ 25,210
Total Cash Flows (Levered)	2.86%	$ (100,000)	$ 3,242	$ (1,019)	$ (831)	$ (145)	$ 513	$ 3,277	$ 116,347
Use of cash		$ 20,173	$ -	$ -	$ -	$ -	$ -	$ -	$ (25,210)
Total Cash Flows		20,173	23,415	22,396	21,564	21,420	21,932	25,210	116,347
% of Cash to total Value		10.2%	11.8%	11.0%	10.4%	10.2%	10.4%	11.7%	

HPR (Levered) R_p=	21.38%	
Risk Free Rate (R_{fr}) =	1.00%	(7-month Interpolated Treasury Bill)
Portfolio Standard Deviation (σ_p)=	17.57%	(Levered adjusted)
Sharpe Ratio (SR) =	**1.16**	

Unlevered Return Calculation:	0 June 1 20x1	1 July 1 20x1	2 Aug 1 20x1	3 Sep 1 20x1	4 Oct 1 20x1	5 Nov 1 20x1	6 Dec 1 20x1	7 Jan 2 20x2
Total Cash Flows (Levered)	$ (100,000)	$ 3,242	$ (1,019)	$ (831)	$ (145)	$ 513	$ 3,277	$ 116,347
Addback Loan Principal	(100,000)	-	-	-	-	-	-	100,000
Addback Loan Interest	-	417	417	417	417	417	417	417
Unlevered Cash Flow	$ (200,000)	$ 3,658	$ (603)	$ (415)	$ 272	$ 929	$ 3,694	$ 216,764

HPR (UnLevered) =	12.15%	
Risk Free Rate (R_{fr}) =	1.00%	(7-month Interpolated Treasury Bill)
Portfolio Standard Deviation (σ_p)=	9.98%	
Sharpe Ratio (SR) =	**1.12**	

Figure 4.6 below shows all performance ratios discussed earlier in this chapter for Zeus Fund 1:

Insert Figure 4.6

ZEUS Fund I

Portfolio Perfromance Ratio Analysis (Monthly)

	Description	Symbol	Zeus Stock Portfolio	Stock Benchmark Market (m)	Zeus Bond Portfolio	Bonds Benchmark Market (m)	Zeus Combined Portfolio	Weighted Benchmark Market (m)
INPUT	Average Montly Return	*R*	3.38%	1.22%	0.54%	0.15%	1.86%	0.81%
	Risk Free Return (Monthly)	*Rf*	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%
	Standard Deviation	σ	2.80%	1.17%	0.75%	0.46%	0.02%	0.43%
	Beta	β	1.825x	1.000x	1.124x	1.000x	1.446x	1.000x
OUTPUT	Risk Premium Return	*RPR*	3.23%	1.08%	0.40%	0.01%	1.72%	0.66%
	Market Premium	*Pm*	1.08%	1.08%	0.01%	0.01%	0.66%	0.66%
	Capital Asset Pricing Model	*CAPM*	2.11%	1.22%	0.15%	0.15%	1.10%	0.81%
	Sharpe Ratio	*SR*	1.156	0.923	0.535	0.017	92.745	1.556
	Jensen's Alpha	α	1.270%	0.000%	0.391%	0.000%	0.758%	0.000%
	Treynor Measure	*T*	1.772%	1.077%	0.356%	0.008%	1.188%	0.664%
	M-Square	M^2	1.492%	1.220%	0.390%	0.151%	39.728%	0.807%

Portfolio Perfromance Ratio Analysis (Annualized)

	Description	Symbol	Zeus Stock Portfolio	Stock Benchmark Market (m)	Zeus Bond Portfolio	Bonds Benchmark Market (m)	Zeus Combined Portfolio	Weighted Benchmark Market (m)
INPUT	Average Annualized Return	*R*	40.53%	14.64%	6.51%	1.81%	22.34%	9.68%
	Risk Free Return (Annualized	*Rf*	1.71%	1.71%	1.71%	1.71%	1.71%	1.71%
	Standard Deviation	σ	33.57%	14.00%	8.97%	5.54%	0.22%	5.12%
	Beta	β	1.825x	1.000x	1.124x	1.000x	1.446x	1.000x
OUTPUT	Risk Premium Return	*RPR*	38.82%	12.92%	4.80%	0.09%	20.62%	7.97%
	Market Premium	*Pm*	12.92%	12.92%	0.09%	0.09%	7.97%	7.97%
	Capital Asset Pricing Model	*CAPM*	25.30%	14.64%	1.82%	1.81%	13.24%	9.68%
	Sharpe Ratio	*SR*	1.156	0.923	0.535	0.017	92.745	1.556
	Jensen's Alpha	α	15.235%	0.000%	4.695%	0.000%	9.096%	0.000%
	Treynor Measure	*T*	21.269%	12.922%	4.271%	0.093%	14.258%	7.969%
	M-Square	M^2	3.063%	2.791%	1.961%	1.722%	41.300%	2.378%

Figure 4.6

CASE STUDY AND PRACTICE CASES

1. Based on the information below, complete the projected spreadsheet. (access spreadsheet www.professordrou.com)

   TO BE PROVIDED LATER

References (Chapter 4)

TO BE PROVIDED LATER