Chapter 2
Covariance, Correlation and Efficient Frontiers

The last chapter established the basics for measuring risk and quantifying the expected return, as well introduced few ways to get increased efficiency out of this relationship through allocation and diversification. This chapter will continue to focus on the return, risk, their respective relationship and allocation. This chapter will further discuss the correlation between two assets which is the primary reason for achieving higher portfolio efficiency. The covariance and correlation analysis are the factors that contribute to at minimum variance and achieve an efficient frontier.

Learning Objectives

After reading this chapter, students will be able to:

- Compute the covariance and correlation between two asset classes such as stocks and bonds.
- Understand diversification benefits and explain how the correlation in a two-asset portfolio affect the diversification benefits.
- Quantify the weights between two asset classes, such as stock and bonds, to achieve the efficient frontier
- Understand the sensitivity of a combined portfolio at different correlation levels between minus 1.0 and positive 1.0.
- Use a sample portfolio example that consists of stocks and bonds to show how the standard deviation of this combined portfolio changes at different correlations.

Covariance & Correlation Overview

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AUTHOR'S NOTES:
Correlation between two unrelated events or two independent objects has been a focus on a lot of scientists, statisticians and others that try to find common ground to justify their dependence or non-dependence. It does remind me of a story that I heard long time ago. The story goes like this: The native American chief of a tribe somewhere in upstate Massachusetts, in preparation for the upcoming winter and plan how much wood the tribe needs to collect, approached the tribe's wise man to ask him what his prediction on how the coming winter will be. For the first time the wise man was not getting any signs using his intuition to make such prediction. The chief then, for the first time, called the American Meteorological Association to find out if they had run studies to predict the temperatures for the upcoming winter. According to them they were predicting colder than usual temperatures. The chief then asked all the tribesmen to start gathering wood. A week

later, to make sure, the chief called again, and they said that the new prediction is that this winter will definitely be colder than past winters. The chief then asked his staff to go back and cut more wood as it will be a very cold winter. After a week, the chief made one more call to make sure what they were predicting was right. He asked them what measurement methods they were using to predict such low temperatures. Someone from the American Meteorological Society said "Well, it will be the coldest winter ever….you should see how much wood the natives are gathering this year." Of course, this is a funny way of addressing correlation. It highlights when each event dependents on each other to the point that the self-prophecy will end up happening. It is felt that is always the case between the Fed raising rates and the stock market. The stock market reacts to the Fed's comments of raising or decreasing rates but sometimes the Fed chairman, based on human nature, could influence his or her decision. Therefore, the market reaction is not unexpectedly surprising and can trigger other events.

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KEY TAKEAWAYS:

- *Covariance and Correlation between two assets which measured between minus 1 and positive 1. This is the cause of how two standalone portfolios with historical risks and returns when combined can achieve better than average risk.*
- *At perfect positive correlation – basically positive 1 – between two asset classes or two portfolios will not allow for any efficiencies if the investor combines these assets or portfolios since the risk and return are moving at the same direction.*
- *At perfect negative correlation – basically negative 1 – between two asset classes or two portfolios will almost definitely allow for efficiencies if the investor combines these assets or portfolios. In this case, the investor could benefit from the average return between the two asset classes or portfolios and at the same time minimize the standalone standard deviations to a lower combined one*
- *The relationship between average return and combined standard deviation is the basis for introducing the concept of the Sharpe Ratio that will be discussed in later chapters.*

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Covariance and Correlation Overview

Investment Return and Risk Efficiency

As it was mentioned in the first chapter, the four factors that an investor needs to consider before investing is to ***first*** measure the return expectation, ***second***, quantify the risk or the probability to achieve such expectation. The ***third*** factor is to seek the right allocation of investments to various asset classes not only to diversify the risk but also to possibly achieve higher efficiencies and ***fourth*** is to set the time to achieve such goal or establish the exit strategy. The previous chapter touched on the first two factors. This chapter and many chapters to follow will always continue to emphasize the first two factors and the importance of the relationship between risk and return. This Chapter will focus more on the third and fourth factors. All four factors are essential to build expectation. The investor needs to determine return expectation first. Without any expectations the investor should not proceed with the investment. The expectation comes with risk appetite. The basic premise of course is that the higher the risk the higher the return expectation. We will examine a possibility that when the investor allocates his/her investments across different assets will not only diversify the risk but can also achieve efficiency or attempt to better balance the risk with the return.

The investment thesis is based on the idea that by diversifying or allocating your investments to various assets classes can achieve higher efficiency. This is the point where the investment has the highest possible return at the lowest possible risk or volatility. The single biggest factor that drives such efficiency is correlation. The key assessment of portfolio risk is the measurement of which the returns on two asset classes or more move in tandem or opposite. Portfolio risk depends on the correlation between the returns of the investments in the portfolio.

To construct a portfolio of risky assets, the portfolio manager needs to understand how the uncertainties can affect the returns of these assets. Starting with the example in chapter 1 the following figure (figure 2.1) shows that the standard deviation of the combined portfolio that has 60% stocks and 40% bonds of 6.24% is lower than an -all bond portfolio of 7.05%. This phenomenon is credited to the negative correlation shown on figure 2.1 of -0.97.

Covariance & Correlation Calculations – the specifics

Let's compare two asset classes such as stocks and bonds as illustrated in figure 2.1. After we calculate each deviation from their respective mean for each economic scenario and apply the probability, we will then calculate the covariance Cov (Rs, Rb) for stock and bonds which is the sum of these deviations. The Correlation is calculated as follows:

$$\rho = \frac{\mathrm{cov}(R_s R_b)}{\sigma s \cdot \sigma_b}$$

Insert Figure 2.1

EFFICIENCY THROUGH CORRELATION

SCENARIO PERFROMANCE ANALYSIS

		Stocks (s)					Bonds (b)				
Scenario (S)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD	ROR % (r_b)	p * r_b %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD
Recession (Sr)	25.0%	-12.00	-3.00	-23.70	561.69	140.42	14.00	3.50	9.75	95.06	23.77
Normal (Sn)	45.0%	14.00	6.30	2.30	5.29	2.38	5.00	2.25	0.75	0.56	0.25
Boom (Sb)	30.0%	28.00	8.40	16.30	265.69	79.71	-5.00	-1.50	-9.25	85.56	25.67
	100.0%		11.70 %		Variance=	222.51		4.25 %		Variance=	49.69
					SD =	14.92 %				SD =	7.05 %

PORTFOLIO ANALYSIS (Asset Allocation)

Asset Allocation	Weights (W%)
Stocks (Ws) =	60%
Bonds (Wb) =	40%

(Ws * r_s) + (Wb * r_b)

COVARIANCE & CORRELATION

Scenario (S)	Probability (p)	ROR % (r_s)	p * r_s %	Deviation for Exp. Ret. (Dev.)	Square Deviation (SD) Dev^2	p * SD	Stocks (Deviation from the mean)	Bonds (Deviation from the mean)	Ds * Db	Covariance [p * (Ds*Db)]
Recession (Sr)	25.0%	-1.6	-0.40	-10.32	106.50	26.63	-23.70	9.75	-231.08	-57.77
Normal (Sn)	45.0%	10.4	4.68	1.68	2.82	1.27	2.30	0.75	1.73	0.78
Boom (Sb)	30.0%	14.8	4.44	6.08	36.97	11.09	16.30	-9.25	-150.78	-45.23
	100.0%		8.72 %		Variance=	38.99			Covarianc	-102.23
					SD =	6.24 %			Correlation Coefficient =	-0.97

Figure 2.1

The Basics of Risk-Return (Mean-Variance) Analysis

Tough the risk return relationship will be discussed in depth in the unit of this book that will cover portfolio analysis, it is necessary to establish a basis for choosing the right portfolio. **The mean-variance portfolio theory provides the theoretical foundation for assessing the risk return relationship when selecting a portfolio**. Mean-variance portfolio risk theory is based on the concept that the value of a specific investment can be measured in terms of mean return and variance return. The analysis first assumes that the investor will always seek higher return at lower risk (taking out the behavioral aspect of investing) as we discussed on chapter one and further prove the point in this chapter. Also, the analysis assumes that the expected return, variance and covariances are already quantified based on historical information. Once the information is understood and accepted as the basis of historical assessment, the analysis will then use that to build the investment expectation and run various sensitivities to identify the efficient and optimum levels.

The impact of Correlation to portfolio efficiency – achieving minimum variance

When combining two asset class in one portfolio, the combined return, variance and standard deviation can be achieved as follows:

Mean Return (Average Return)

$$R_\rho = (w_s \cdot R_s) + (w_b \cdot R_b)$$

Where R_ρ is the return of the combined portfolio, R_s is the return of the stock portfolio, R_b is the return of the bond portfolio and w_s and w_b are the percentage weights of stock and bonds respectively.

Variance and Standard Deviation

$$\sigma_P^2 = w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\rho$$

$$\sigma_P = \sqrt{(}w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s\sigma_s w_b\sigma_b\rho)$$

Where $\boldsymbol{\sigma_P^2}$ is the variance of the combined portfolio, w_s and w_b are the percentage weights of stocks and bonds, respectively, $\boldsymbol{\sigma_s}$ and $\boldsymbol{\sigma_b}$ are the standard deviation of the stocks and bonds, respectively and $\boldsymbol{\rho}$ is the correlation.

Correlation, expressed with the Greek rho $(\boldsymbol{\rho})$ **,** can significantly change the portfolio's variance and standard deviation.

Before we examine the impact of correlation to portfolio efficiency, lets discuss what is Efficiency and express the concept by using an example of two portfolios. As shown figure 2.1 above by trading out 100% bonds to 60% stocks and 40% bonds, the bond portfolio standard deviation has improved from 7.05% to 6.24%. Then graph 2.2. below shows the capital allocation line as you move from 100% bonds to 100% stock, traveling left before the line turns right towards the stock. **The most northwester point of the map before the turn is the efficient frontier**. **This is the point with the highest possible return at the lowest possible risk measured by the standard deviation.**

Insert Figure 2.2

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	-0.972

			PORTF. A CORRELATION	
Correlation =			**-0.972**	
Portfolio Weights			**Standard Deviation**	**Expected Return**
W% stocks	**W% bonds**		**σ %**	**E(r) %**
0%	100%		7.05	4.25
10%	90%		4.91	5.00
20%	80%		2.83	5.74
30%	70%	Efficiency	1.20	6.49
40%	60%		2.10	7.23
50%	50%		4.12	7.98
60%	40%		6.24	8.72
70%	30%		8.40	9.47
80%	20%		10.57	10.21
90%	10%		12.74	10.96
100%	0%		14.92	11.70

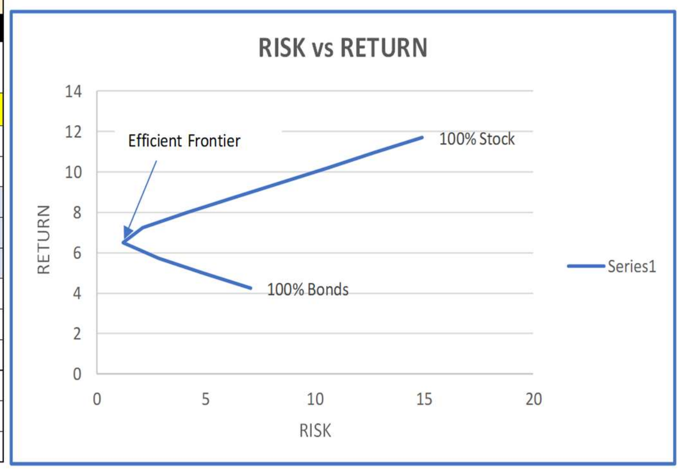

Figure 2

Efficient Frontier at different correlation levels.

Assuming we take the portfolio example of risk return characteristics above but use different levels of correlation between negative (-1) to positive (+1). The portfolio efficiency dramatically changes from the highest level to no efficiency. **A portfolio with perfect positive correlation does not yield any efficiency as the portfolio manager moves from bonds to stock.** A portfolio with perfect negative efficiency could reach the maximum – conceivably eliminating any risk as the portfolio trades out of a bond to a stock portfolio. At zero correlation the portfolio continues to be tested for any efficiencies. The chart below (Figure 2.3) shows all three possibilities and compares to the portfolio example used above.

Insert Figure 2.3

FINDING RISK RETURN EFFICIENCY (EFFICIENT FRONTIER)

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	-0.972

			ZERO CORRELATION		POSITIVE CORRELATION		NEGATIVE CORRELATION		PORTF. A CORRELATION	
Correlation =			0.000		1.000		-1.000		-0.972	
Portfolio Weights			Risk	Return	Risk	Return	Risk	Return	Risk	Return
W% stocks	W% bonds		σ %	E(r) %	σ %	E(r) %	σ %	E(r) %	σ %	E(r) %
0%	100%		7.05	4.25	7.05	4.25	7.05	4.25	7.05	4.25
10%	90%	Efficiency	6.52	5.00	7.84	5.00	4.85	5.00	4.91	5.00
20%	80%		6.38	5.74	8.62	5.74	2.66	5.74	2.83	5.74
30%	70%		6.66	6.49	9.41	6.49	0.46	6.49	1.20	6.49
40%	60%		7.31	7.23	10.20	7.23	1.74	7.23	2.10	7.23
50%	50%		8.25	7.98	10.98	7.98	3.93	7.98	4.12	7.98
60%	40%		9.38	8.72	11.77	8.72	6.13	8.72	6.24	8.72
70%	30%		10.65	9.47	12.56	9.47	8.33	9.47	8.40	9.47
80%	20%		12.02	10.21	13.34	10.21	10.52	10.21	10.57	10.21
90%	10%		13.44	10.96	14.13	10.96	12.72	10.96	12.74	10.96
100%	0%		14.92	11.70	14.92	11.70	14.92	11.70	14.92	11.70

Figure 2.3

The charts below (Figures 2.4, 2.5 and 2.6) shows the 2-asset class capital allocation line from the extreme bend to the left representing the perfect negative correlation between bond and stocks to a straight line representing no efficiency – **basically, as the portfolio manager trade out of bonds and substitutes it with equity to achieve a higher return, since the correlation between these two assets are a perfect 1 the additional return you get will come with the exact linear additional risk.**

Scenario Analysis Example

Figure 2.4 below shows our portfolio (portfolio A) with an assume zero correlation. The efficiency can be achieved around 10%-20% stock allocation showing the standard deviation at these levels is reduced from 7.05% all (bonds) to 6.52% at 10% Stock and continue to reduce to 6.38% at 20% stock before the standard deviation increases again around 30% showing a standard deviation of 6.66%

Insert Figure 2.4

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	0.000

			ZERO CORRELATION	
Correlation =			**0.000**	
Portfolio Weights			**Risk**	**Return**
W% stocks	**W% bonds**		**σ %**	**E(r) %**
0%	100%		7.05	4.25
10%	90%		6.52	5.00
20%	80%	Efficiency	6.38	5.74
30%	70%		6.66	6.49
40%	60%		7.31	7.23
50%	50%		8.25	7.98
60%	40%		9.38	8.72
70%	30%		10.65	9.47
80%	20%		12.02	10.21
90%	10%		13.44	10.96
100%	0%		14.92	11.70

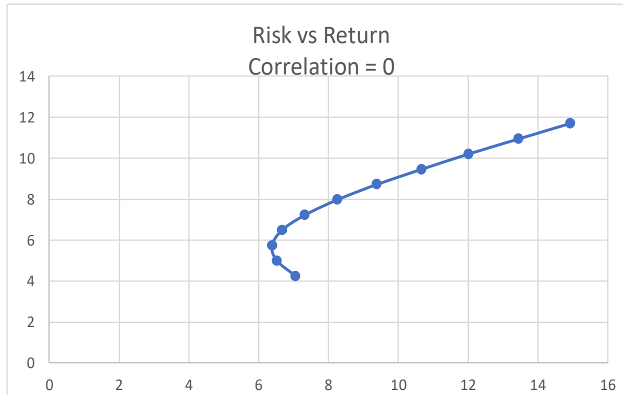

Figure 2.4

Figure 2.5 below shows our portfolio's (portfolio A) risk versus return allocation line assuming a perfect positive +1 correlation. At a +1 correlation there is no efficiency. As the portfolio moves from all bonds to all stock the line is at 45-degree angle showing that the risk continues to increase at the same pace as the portfolio manager is seeking higher returns.

Insert Figure 2.5

Portfolio A

E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	1.000

		POSITIVE CORRELATION	
Correlation =		**1.000**	
Portfolio Weights		**Risk**	**Return**
W% stocks	**W% bonds**	**σ %**	**E(r) %**
0%	100%	7.05	4.25
10%	90%	7.84	5.00
20%	80%	8.62	5.74
30%	70%	9.41	6.49
40%	60%	10.20	7.23
50%	50%	10.98	7.98
60%	40%	11.77	8.72
70%	30%	12.56	9.47
80%	20%	13.34	10.21
90%	10%	14.13	10.96
100%	0%	14.92	11.70

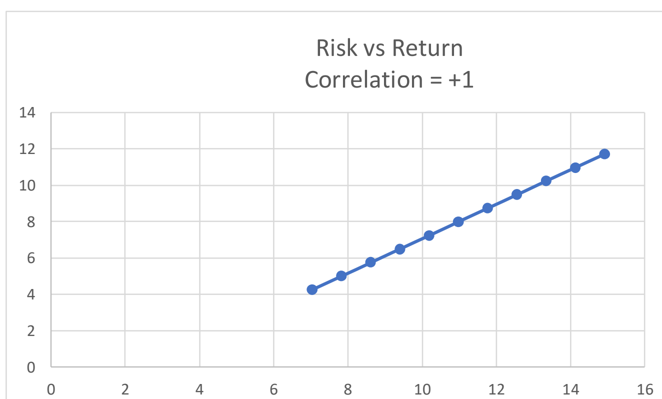

Figure 2.5

Figure 2.6 below shows our portfolio (portfolio A) with perfect negative -1 correlation. At -1 correlation since the two assets held at the portfolio are moving in the opposite way from each, the risk can be offset as the portfolio manager is moving from an all bond portfolio to all stock. The chart shows that the efficient frontier, which is the point where the combined portfolio of bonds and stocks have the highest possible return at the lowest possible risk is at efficiency can be achieved around 30% stock and 70% bonds showing a combined return of 6.49% at 0.46% standard deviation.

Insert Figure 2.6

Portfolio A	
E (rs) =	11.700
E (rb) =	4.250
σs =	14.917
σb =	7.049
Correlation=	-1.000

			NEGATIVE CORRELATION	
Correlation =			-1.000	
Portfolio Weights			Risk	Return
W% stocks	W% bonds		σ %	E(r) %
0%	100%		7.05	4.25
10%	90%		4.85	5.00
20%	80%		2.66	5.74
30%	70%	Efficiency	0.46	6.49
40%	60%		1.74	7.23
50%	50%		3.93	7.98
60%	40%		6.13	8.72
70%	30%		8.33	9.47
80%	20%		10.52	10.21
90%	10%		12.72	10.96
100%	0%		14.92	11.70

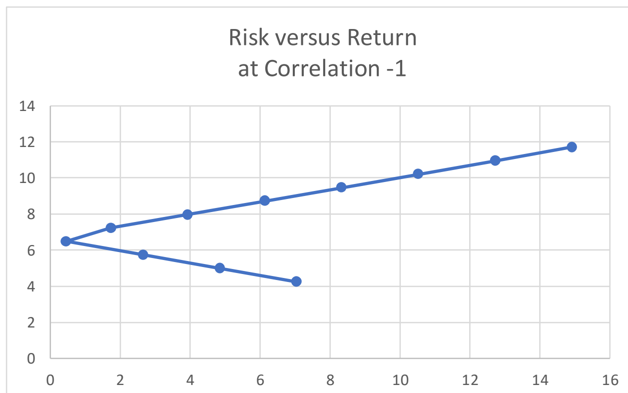

Figure 2.6

Historical Analysis Example

Figure 2.7 below shows a portfolio consisting of 12-year historical information of stocks and bonds. The average return and standard deviation for the stocks are 11.68% and 19.20% respectively. The bonds on the hand have 2.08% average return and 4.97% standard deviation. The covariance and correlation between stocks and bonds is at -70.05 and negative -0.73x respectively (please note that the average variance is calculated by 11 observations and not 12, hence the denominator is n-1 rather n). This is because the deviations are calculated based on estimated average return for that year instead of the true expected return. In statistics this adjustment, called “degrees of freedom”, is to accommodate for any unknown errors in the information. In Excel, the function is =stdev.s for the n-1 instead of the excel function of =stdev.p or =stdev which is divided by n observations. We also run a combined portfolio with 30% stocks and 70% bonds showing a higher return and lower standard deviation 4.96% and 3.99%, respectively as compared to all bond portfolio with return and standard deviation of 2.08% and 4.97%.

Insert Figure 2.7

HISTORICAL ANALYSIS

	Returns		Deviations from Average Return		Standard Deviation		Product from Deviation
	Stocks %	Bonds %	Stocks (Ds)	Bonds (Db)	Stocks	Bonds	Ds . Db
Year -12	-6.50	3.10	-18.18	1.02	330.33	1.03	-18.48
Year -11	-13.20	5.20	-24.88	3.12	618.77	9.71	-77.53
Year -10	-8.90	7.90	-20.58	5.82	423.33	33.83	-119.68
Year -9	25.00	6.10	13.33	4.02	177.56	16.13	53.52
Year -8	48.50	-9.50	36.83	-11.58	1356.08	134.17	-426.56
Year -7	37.60	-2.50	25.93	-4.58	672.11	21.01	-118.82
Year -6	10.50	2.50	-1.18	0.42	1.38	0.17	-0.49
Year -5	7.20	1.50	-4.48	-0.58	20.03	0.34	2.61
Year -4	-5.60	3.40	-17.28	1.32	298.43	1.73	-22.75
Year -3	17.50	-3.20	5.83	-5.28	33.93	27.91	-30.78
Year -2	21.50	3.50	9.83	1.42	96.53	2.01	13.92
Year -1	6.50	7.00	-5.18	4.92	26.78	24.17	-25.44

	Stocks	Bonds
Average Return	**11.68**	**2.08**
Standard Deviation	19.20	4.97
Covariance	-70.04	
Correlation	-0.73	

	Stocks	Bonds		
Total	**4055.24**	**272.24**		**-770.47**
Average (use n-1)	368.66	24.75	Cov=	-70.04
Standard Deviation	19.20	4.97	Correl=	-0.73

Combinced Portfolio at 30% Stocks and 70% Bonds	
Average Return	**4.96**
Standard Deviation	**3.99**

Figure 2.7

[Insert boxed text here

Excel formulas for Average, Standard Deviation, Covariance and Correlation:

= Average (number1, number2,….) – highlight information Range
=Stdev.p (number1, number 2,…) for n observations, =stdev.s for n-1 observations
=Covar (aray1, array2) – highlight each comparative range
=Correl(array1, array2) - highlight each comparative range

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Allocating various percentages between stocks bonds can achieve efficiency. Figure 2.8 shows that the efficiency (lowest possible combined standard deviation) is between the 10-30% stocks and 70-90% bonds.

Insert Figure 2.8

HISTORICAL ANALYSIS

Portfolio Consruction

	W% Stocks	W% Bonds	Weighted Average St. Dev.	Weighted Average Return
	0%	100%	4.97	2.08
	10%	90%	3.14	3.04
Efficiency	20%	80%	2.44	4.00
	30%	70%	3.60	4.96
	40%	60%	5.56	5.92
	50%	50%	7.73	6.88
	60%	40%	9.98	7.84
	70%	30%	12.27	8.80
	80%	20%	14.57	9.76
	90%	10%	16.88	10.72
	100%	0%	19.20	11.68

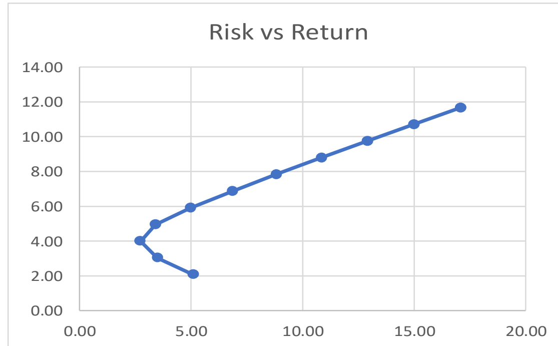

Figure 2.8

Extension to the Three-Asset Case

Earlier in this chapter we discussed how to form a two-asset class portfolio consist of stocks and bonds and how to allocate these assets, so the investor can achieve efficiency. Let's assume that the investor is now seeking to continue maximizing the expected return with the lowest possible minimum volatility by adding another asset class in the portfolio. **The question is how the investor will could improve the trade-off between risk and return by adding a new asset class in the portfolio.** Figure 2.9 below shows the existing portfolio from figure 2.7 which consists of large-cap stocks and corporate bonds and trade out of these to buy a portfolio consisting of small-cap stocks.

Figure 2.9 shows that the investor first holds 30% large-cap stocks and 70% bonds that achieves a more efficient portfolio if the investor had only bonds. As expected, the return goes up from 2.08% at 100% bonds to 4.96% at 30% stock and 70% bonds. Despite the increase in return, by adding stock to the portfolio of bonds the standard deviation goes down from 4.97% at 100% bonds to 3.99% at 30% stocks and 70% bonds. Now let's add a third asset class such small cap stocks where the 12-year average return and standard deviation is at 13.16% and 23.18% respectively. Obviously, the initial expectation is that by adding a more volatile asset class at 23.18% will increase the risk for the 2-asset class even if we are bringing a much higher return performance from the small-cap stocks. At 10% large-caps stocks, 50% bonds and 40% small-caps stocks, as expected, the return goes up to 7.47% but the standard deviation goes down even further than holding all bonds at 2.0% - achieving further efficiency.

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2-Asset Mean Ret.: $\boldsymbol{R_p = (w_{s1} \cdot R_s) + (w_b \cdot R_b)}$

3- Asset Mean Ret. $\boldsymbol{R_p = (w_{s1} \cdot R_s) + (w_b \cdot R_b) + (w_{s2} \cdot R_{s2})}$

2-Asset St. Dev.: $\boldsymbol{\sigma_P = \sqrt{(w_{s1}^2\sigma_{s1}^2 + w_b^2\sigma_b^2 + 2w_{s1}\sigma_{s1}w_b\sigma_b\rho_{s1b})}}$

3-Asset St. Dev: $\boldsymbol{\sigma_P = \sqrt{(w_{s1}^2\sigma_{s1}^2 + w_b^2\sigma_b^2 + w_{s2}^2\sigma_{s2}^2 + 2w_{s1}\sigma_{s1}w_b\sigma_b\rho_{s1b} + 2w_{s2}\sigma_{s2}w_b\sigma_b\rho_{s2b} + 2w_{s2}\sigma_{s2}w_{s1}\sigma_{s1}\rho_{s1s2})}}$

Where $\boldsymbol{\sigma_P}$ is the standard deviation of the combined portfolio, w_{s1} , w_{s2} and w_b are the percentage weights of large-cap stocks, small-cap stocks and bonds, respectively, $\boldsymbol{\sigma_{s1}}$, and $\boldsymbol{\sigma_{s2}}$and $\boldsymbol{\sigma_b}$ are *the standard deviations of the large-cap stocks, small-cap stocks and bonds, respectively and* ρ_{s1b} *,* ρ_{s2b} *and* ρ_{s1s2} *are the correlations.*

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Insert Figure 2.9

THREE-ASSET CASE

Achieving efficiency by adding a third asset class

	Returns		
	Large-Cap Stocks %	**Bonds %**	**Small-Cap Stocks %**
Year -12	-6.50	3.10	-7.80
Year -11	-13.20	5.20	-16.00
Year -10	-8.90	7.90	-11.00
Year -9	25.00	6.10	21.00
Year -8	48.50	-9.50	57.00
Year -7	37.60	-2.50	49.00
Year -6	10.50	2.50	16.50
Year -5	7.20	1.50	9.00
Year -4	-5.60	3.40	-9.60
Year -3	17.50	-3.20	15.00
Year -2	21.50	3.50	27.00
Year -1	6.50	7.00	7.80
Average Return	**11.68**	**2.08**	**13.16**
Standard Deviation	**19.20**	**4.97**	**23.18**
% Holdings before Extension	30.0%	70.0%	
% Holdings including new Extension	10.0%	50.0%	40.0%

Correlation	
Large-Cap Stocks and Bonds	-0.733
Small Cap-Stocks and Large Cap Stocks	0.987
Bond and Small Cap-Stocks	-0.738

Portfolio Results	
Return for 2-Asset Holdings	4.96
Standard Deviation for 2-Asset Holdings	**3.99**
Return for 2-Asset Holdings	7.47
Standard Deviation for 3-Asset Holdings	**2.00**

Figure 2.9

Portfolio of Stocks and Bonds – Zeus Fund I - Case Study

The previous chapter describes the initial capital raised to fund a newly established $200 million fund called Zeus Fund I. Zeus Fund I is set-up to buy stock and corporate bonds as shown Chapter 1, figure 1.7.

Figures 1.11 and Figures 1.12 from Chapter 1 shows all the stock and bond trades over 7 months. Figure 2.10 below calculates the average monthly changes of the Zeus Fund I portfolio of stocks and bonds.

Insert Figure 2.10

ZEUS Fund I

STOCK PORTFOLIO

Stock Prices		0	1	2	3	4	5	6	7
Symbol	**Company Name**	**June 1 20x1**	**July 1 20x1**	**Aug 1 20x1**	**Sep 1 20x1**	**Oct 1 20x1**	**Nov 1 20x1**	**Dec 1 20x1**	**Jan 2 20x2**
ABC	ABC Chem Inc	23.00	24.00	22.50	25.00	26.00	27.00	28.00	31.00
BCD	BCD Precision Inc	12.00	10.00	12.00	12.00	15.00	18.00	19.50	22.00
CDE	CDE Inc	18.00	19.00	18.00	19.00	21.00	20.00	19.00	21.00
DEF	DEF Inc	40.00	42.00	43.00	45.00	45.00	45.00	46.00	48.00
EFG	Effective Inc	52.00	60.00	60.00	60.00	62.00	62.00	61.00	63.00
FGH	FGH Inc	31.00	20.00	25.00	26.00	20.00	22.00	24.00	25.00
GHI	General HI	15.00	16.00	17.00	18.00	19.00	19.00	18.00	20.00
HIK	Hicks Kental Inc	8.00	9.50	10.50	11.00	11.50	12.00	14.00	14.50
IKL	IKL Inc	15.00	13.00	12.00	14.00	15.00	18.00	22.00	20.00
KLM	KLM Health	25.00	26.00	26.00	26.00	26.00	26.00	27.00	20.00
LMN	LMN Hotel & Resorts	26.00	30.00	32.00	33.00	35.00	32.00	34.00	35.00
MNO	MNO Cable Inc	19.00	20.00	19.00	18.00	18.00	16.00	20.00	18.00
NOP	Norton Optimum	53.00	52.00	55.00	56.00	58.00	59.00	59.00	61.00
OPQ	Odyssea PQ Inc	11.00	8.50	11.00	11.00	11.00	11.00	11.50	12.00
PQR	PQR Chemicals	18.00	17.00	19.00	19.00	20.00	22.00	26.00	24.00

Stock Prices		0	1	2	3	4	5	6	7
Symbol	**Company Name**	**June 1 20x1**	**July 1 20x1**	**Aug 1 20x1**	**Sep 1 20x1**	**Oct 1 20x1**	**Nov 1 20x1**	**Dec 1 20x1**	**Jan 2 20x2**
ABC	ABC Chem Inc		4.3%	-6.3%	11.1%	4.0%	3.8%	3.7%	10.7%
BCD	BCD Precision Inc		-16.7%	20.0%	0.0%	25.0%	20.0%	8.3%	12.8%
CDE	CDE Inc		5.6%	-5.3%	5.6%	10.5%	-4.8%	-5.0%	10.5%
DEF	DEF Inc		5.0%	2.4%	4.7%	0.0%	0.0%	2.2%	4.3%
EFG	Effective Inc		15.4%	0.0%	0.0%	3.3%	0.0%	-1.6%	3.3%
FGH	FGH Inc		-35.5%	25.0%	4.0%	-23.1%	10.0%	9.1%	4.2%
GHI	General HI		6.7%	6.3%	5.9%	5.6%	0.0%	-5.3%	11.1%
HIK	Hicks Kental Inc		18.8%	10.5%	4.8%	4.5%	4.3%	16.7%	3.6%
IKL	IKL Inc		-13.3%	-7.7%	16.7%	7.1%	20.0%	22.2%	-9.1%
KLM	KLM Health		4.0%	0.0%	0.0%	0.0%	0.0%	3.8%	-25.9%
LMN	LMN Hotel & Resorts		15.4%	6.7%	3.1%	6.1%	-8.6%	6.3%	2.9%
MNO	MNO Cable Inc		5.3%	-5.0%	-5.3%	0.0%	-11.1%	25.0%	-10.0%
NOP	Norton Optimum		-1.9%	5.8%	1.8%	3.6%	1.7%	0.0%	3.4%
OPQ	Odyssea PQ Inc		-22.7%	29.4%	0.0%	0.0%	0.0%	4.5%	4.3%
PQR	PQR Chemicals		-5.6%	11.8%	0.0%	5.3%	10.0%	18.2%	-7.7%
Average Return			-1.0%	6.2%	3.5%	3.5%	3.0%	7.2%	1.2%

BOND PORTFOLIO

Bond Prices		0	1	2	3	4	5	6	7
Symbol	**Company Name**	**June 1 20x1**	**July 1 20x1**	**Aug 1 20x1**	**Sep 1 20x1**	**Oct 1 20x1**	**Nov 1 20x1**	**Dec 1 20x1**	**Jan 2 20x2**
AAA	Alpha Inc.	890	893	895	905	910	912	915	910
BBB	Beta Inc.	910	925	915	925	915	922	935	930
CCC	CC Corporation	790	800	810	815	820	822	815	800
DDD	Delta D Inc.	1010	1015	1020	1022	1026	1025	1020	1027
EEE	Epsilon Inc	950	965	975	980	982	995	1000	1010
FFF	Fusbol For Friends	640	680	687	695	710	720	710	700

Bond Prices Monthly %		0	1	2	3	4	5	6	7
Symbol	**Company Name**	**June 1 20x1**	**July 1 20x1**	**Aug 1 20x1**	**Sep 1 20x1**	**Oct 1 20x1**	**Nov 1 20x1**	**Dec 1 20x1**	**Jan 2 20x2**
AAA	Alpha Inc.		0.34%	0.22%	1.12%	0.55%	0.22%	0.33%	-0.55%
BBB	Beta Inc.		1.65%	-1.08%	1.09%	-1.08%	0.77%	1.41%	-0.53%
CCC	CC Corporation		1.27%	1.25%	0.62%	0.61%	0.24%	-0.85%	-1.84%
DDD	Delta D Inc.		0.50%	0.49%	0.20%	0.39%	-0.10%	-0.49%	0.69%
EEE	Epsilon Inc		1.58%	1.04%	0.51%	0.20%	1.32%	0.50%	1.00%
FFF	Fusbol For Friends		6.25%	1.03%	1.16%	2.16%	1.41%	-1.39%	-1.41%
Average Return			1.9%	0.5%	0.8%	0.5%	0.6%	-0.1%	-0.4%

Figure 2.10

Figure 2.11 below shows the overall combined stock/bond 7-month performance of Zeus Fund I portfolio. The average portfolio made of 46.4% stocks and 53.6% bonds yield an average rate or return (ROR) of 1.86% per month with 1.1415% volatility. The correlation is calculated at -0.521. Finding the efficiency based on the historical performance is at approximately 10-20% stocks and 80-90% bonds. At these levels the standard deviation is lower than an all bond portfolio.

Insert Figure 2.10

ZEUS Fund I

STOCK AND BOND PORTFOLIO

Performance	0	1	2	3	4	5	6	7
Symbol		July 1 20x1	Aug 1 20x1	Sep 1 20x1	Oct 1 20x1	Nov 1 20x1	Dec 1 20x1	Jan 2 20x1
Stocks		-1.0%	6.2%	3.5%	3.5%	3.0%	7.2%	1.2%
Bonds		1.9%	0.5%	0.8%	0.5%	0.6%	-0.1%	-0.4%
Total Stock Value	81,757	82,600	82,200	84,050	85,000	85,450	87,300	92,275
Total Bonds Value	97,495	95,650	93,620	97,035	101,575	102,110	101,410	97,325
Total Portfolio Value	179,253	178,250	175,820	181,085	186,575	187,560	188,710	189,600
Portfolio % change		-0.6%	-1.4%	3.0%	3.0%	0.5%	0.6%	0.5%
Cummulative % Change		-0.6%	-1.9%	1.1%	4.1%	4.6%	5.2%	5.7%
Stocks (Weights)	45.6%	46.3%	46.8%	46.4%	45.6%	45.6%	46.3%	48.7%
Bonds (Weights)	54.4%	53.7%	53.2%	53.6%	54.4%	54.4%	53.7%	51.3%

7-month Portfolio Performance

STOCKS	
Average	3.3777%
Standard Deviation	2.7976%
Average % of Portfolio	46.3954%
Covariance (Portfolio)	0.000
Correlation (Portfolio)	-0.521

BONDS	
Average	0.5428%
Standard Deviation	0.7477%
Average % of Portfolio	53.6046%

COMBINED PORTFOLIO	
Average	1.8581%
Variance	0.0130%
Standard Deviation	1.1415%
Average % of Portfolio	-0.0093%

Finding Efficiency in the Portfolio

						Current Position						
Stocks	0.000%	10.000%	20.000%	30.000%	40.000%	46.395%	50.000%	60.000%	70.000%	80.000%	90.000%	100.000%
Bonds	100.000%	90.000%	80.000%	70.000%	60.000%	53.605%	50.000%	40.000%	30.000%	20.000%	10.000%	0.000%
Average Return	0.5428%	0.8263%	1.1098%	1.3933%	1.6768%	1.8581%	1.9603%	2.2438%	2.5273%	2.8107%	3.0942%	3.3777%
Standard Deviation	0.7477%	0.5787%	0.5674%	0.7214%	0.9645%	1.1415%	1.2455%	1.5439%	1.8513%	2.1639%	2.4797%	2.7976%

HIGHEST EFFICIENCY

Figure 2.11

NEXT: From Efficiency to Optimization

EFFICIENT FRONTIER → $\frac{\Delta r}{\Delta \sigma} > 1$ → OPTIMIZATION

To review this chapter and set the basis for the following chapters (Chapters 4 and 5), the concept of efficient frontier will be taken further to another level of portfolio assessment. The Efficient frontier works as the starting point of choosing the right investment that enjoys high returns and lowest possible risk. The portfolio manager does not end the search there for building a more effective portfolio. After achieving an efficient portfolio using historical and or scenario analysis, the next move is to go from efficiency to optimization. The optimization, discussed in depth in the following chapters, can be achieved by moving from the efficient frontier to seek additional return at minimum rate of change of risk. As discussed earlier, the efficient frontier is the point where the portfolio manager enjoys the highest possible return to the lowest possible risk. From that point the portfolio analyst is seeking to achieve even a higher return delta (rate of change) but as we discussed it will also come with higher risk. The optimization point is where additional return, or the rate of change going form bonds to stocks should be lower than the rate of change of the additional risk – basically, higher return delta at lower risk delta. That achievement is called Optimum point and the basis of the Sharpe Ratio -discussed in the next few chapters.

CASE STUDY AND PRACTICE CASES

1. Based on the information below, complete the projected spreadsheet. (access spreadsheet www.professordrou.com)

   TO BE PROVIDED LATER

References (Chapter 2)

TO BE PROVIDED LATER