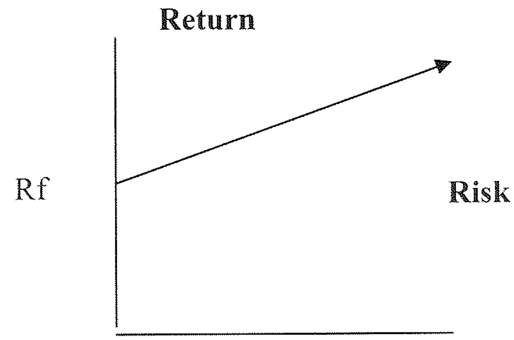
LECTURE 3



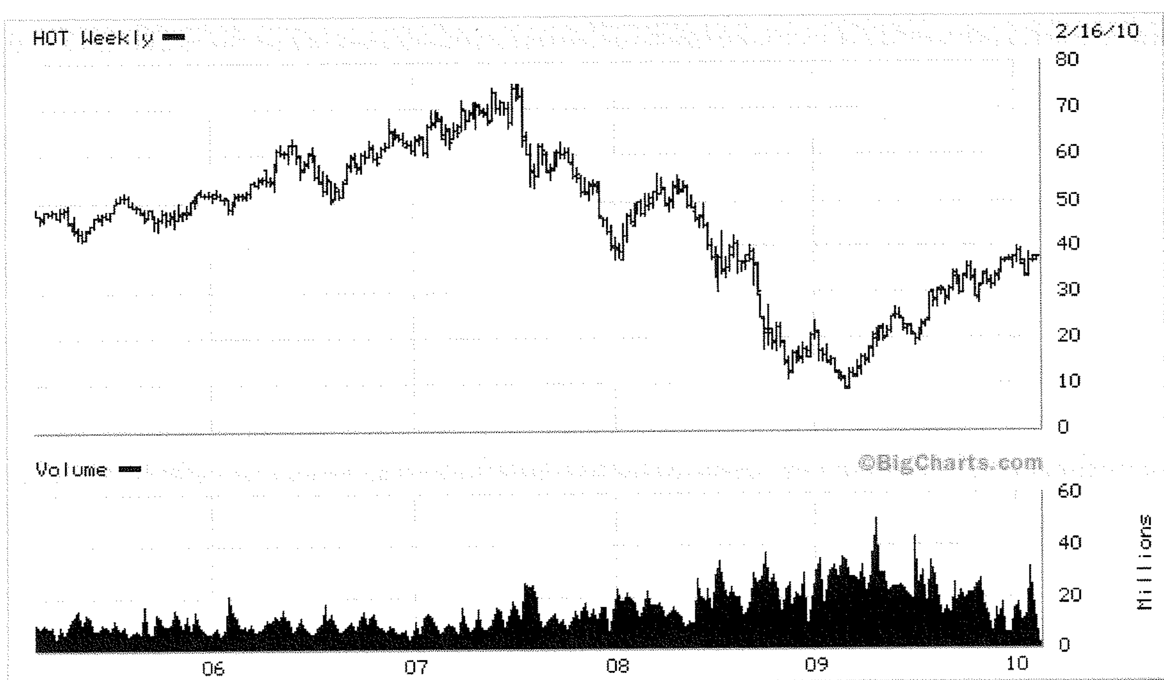
Chapter 5

RISK RETURN –

- Traditionally, when you define return you refer to a bank savings account (risk free) plus a risky portfolio of US stocks. Today, investors have access to a variety of asset classes and financial engineered investments
- The Book "The Quants" by Scott Peterson – financial engineering – achieving the ALPHA.

HPR = (Ending Price – Beg. Price + Div) / Beg. Price

<u>Example:</u>
Current Price = \$100, expected price to increase to \$110 in a year. Within the year you are expected to receive \$4 dividend, therefore the HPR=(110-100+4)/\$100 = 14%



Starwoood Hotels
5-yr stock prices

Measuring Return over a multiple periods

ARITHMETIC AVERAGE (Sum of Quarters)
GEOMETRIC AVERAGE (Single per Quarter – cumulative +compounding)

	0	1	2	3	4
HPR %		10	25	-20	25

Arithmetic:
(10+25-20+25) / 4 = 10%

Geometric:
(1+0.10) * (1+0.25) * (1-0.20) * (1+0.25) = (1+r) ^ 4
r = [(1+0.10) * (1+0.25) * (1-0.20) * (1+0.25)] ^1/4 – 1

(1.1*1.25*.80*1.25)^1/4 – 1 =

1.375 ^1/4 – 1 = 8.29%

Dollar Weighted Return

	0 (initial)	**1**	**2**	**3**	**4**
Net CF ($)	**-1**	**-0.1**	**-0.5**	**0.8**	**1.0**

1 = + (-0.1 / (1+IRR) + (-0.5 / (1+ IRR) ^2 + (0.8 / (1+ IRR) ^3 + (1.0 / (1+IRR) ^4)

Excel

IRR	0	1	2	3	4
4.17%	-1	-0.1	-0.5	0.8	1

=IRR (initial investment, cash flows)

= 4.17%

Conventions for Quoting Rates of Return: Return on Assets with Regular Cash Flow (Mortgage, bonds – semi-annual coupon)

APR (Annual Percentage Rates) – using simple interest approach

APR = per period rate * Periods per Year

EAR (Effective Annual Rate)

1 – EAR = (1 + Rate per period) ^ n = [(1 + (APR/n)) ^n

APR = (1+ EAR) ^1/n -1 * n

For continuous compounding, 1+EAR=e^APR or APR=ln (1+EAR)

$$\mathbf{EAR = (1+Per\ period\ Rate)^{\wedge number\ periods} - 1}$$

Example:

Suppose we buy T-Bills maturing in one month for $9,900 (on maturity you collect the Face Value $10,000).

HPR = (Cash Income + Price Change) / Initial Price

HPR = 100/9,900 = 0.0101 = 1.01%

APR = 1.01% * 12 (annualized) = **12.12%**

EAR → 1 + EAR = (1.0101) ^12 = 1.1282

EAR = 1.1282 – 1 = .1282 = **12.82%**

RISK AND RETURN PREMIUMS

HOW DO WE QUANTIFY RISK?????

Scenarios Analysis and Probability Distributions

	Scenarios	Probability	HPR	WAHPR (probability*HPR)
Boom Growth	1	0.25	44%	11.00%
Normal Growth	2	0.5	14%	7.00%
Recession Growth	3	0.25	-16%	-4.00%
				14.00%

HOW DO WE QUANTIFY THE UNCERTAINTY OF INVESTMENT???

To summarize risk with a single number we find **VARIANCE**, the expected value of the **squared Deviation for the mean,** first. (I.e. the expected value of the squared "surprise": across scenarios.)

$$\text{Var. } (r) = \sigma \wedge 2 = \sum p\,(s)\,[\,r(s) - E\,(r\,)\,)\wedge 2$$

VARIANCE - DEFINITION

The Variance (which is the square of the standard deviation, ie: σ^2) is defined as:

The average of the squared differences from the Mean.

In other words, follow these steps:

1. *Work out the Mean (the simple average of the numbers)*
2. *Now, for each number subtract the Mean and then square the result (the squared difference).*
3. *Then work out the average of those squared differences.*

- Squaring each difference makes them all positive numbers (to avoid negatives reducing the Variance)

- And it also makes the bigger differences stand out. For example 100^2=10,000 is a lot bigger than 50^2=2,500.

- But squaring them makes the final answer really big, and so un-squaring the Variance (by taking the square root) makes the Standard Deviation a much more useful number.

Variance = Squared Sigma

STANDARD DEVIATION **DEFINITION: The Standard Deviation (σ) is a measure of how spreads out numbers are.** (Note: Deviation just means how far from the normal). So, using the

Standard Deviation we have a "standard" way of knowing what is normal, and what is extra large or extra small.

	Scenarios	Probability	HPR	WAHPR (probability*HPR)	Variance [probability*(HPR-sum of WAHPR)^2]
Boom Growth	1	0.25	44%	11.00%	225.00
Normal Growth	2	0.5	14%	7.00%	0.00
Recession Growth	3	0.25	-16%	-4.00%	225.00
				14.00%	450.00
				St. Dev =	**21.21%**

SD (r) = $\sigma = \sqrt{\text{Var}(r)}$

E (r) = (0.25 * 44%) + (0.50 * 14%) + (0.25*-16%) = **14%**

Sigma ^2 = 0.25 (44 – 14) ^2 + 0.50 (14 – 14) ^2 + 0.25 (-16-14)^2 = 450

And so the SD sigma = $\sqrt{450}$ = **21.21%**

EXAMPLE - table 5.2

Current Price=	23.50						
	Scenarios	**Probability**	**End of yr Price**	**Dividends**	**HPR %**	**WAHPR**	**Variance**
High Growth	1	0.35	$ 35.00	$ 4.40	67.66	23.68	591.41
Normal Growth	2	0.30	$ 27.00	$ 4.00	31.91	9.57	8.62
No Growth	3	0.35	$ 15.00	$ 4.00	(19.15)	(6.70)	731.04

E (r) = **26.55** 1,331.07

HPR = (End of the year Price - Current Price + Div) / (Current Price)

StDev = **36.48**

Standard Deviation = Sq Rt of V

Variance = 0.35 * (67.66 - 26.55) ^ 2 + .30 *(31.91 - 26.55) ^ 2 + .35 * (-19.5 - 26.55) ^ 2

RISK PREMIUM Vs RISK AVERSION (Risk Appetite)
We measure the "Reward" or the difference between the expanded HPR or the Index stock fund and the risk – free rate

HPR – Risk Free Rate = Premium

14% - 6 % = 8%

VOLATILITY vs RETURN – Relationship

Sharpe Ratio:

Risk Premium over the Standard Deviation of portfolio excess return

$$(E(r\,p) - r\,f\,) / \sigma$$

8% / 20% = 0.4x. A higher Sharpe ratio indicates a better reward per unit of volatility, in other words, a more efficient portfolio

Sharpe Ratio is more useful for ranking portfolios - it is not valid for individual assets – is useful across Asset Classes.

HISTORICAL RECORD OF RETURNS – TABLE 5.3 – Panel B

To calculate average returns and standard deviations from historical data, let's compute these statistics for the returns on the S&P 500 portfolio using five years of data from the table (5.4)

Example 5.4

Year	ROR	Deviation from Average Return	Deviation From Avg return Squared (^2)
1	16.90%	0.22%	0.05
2	31.30%	14.62%	213.74
3	-3.20%	-19.88%	395.21
4	30.70%	14.02%	196.56
5	7.70%	-8.98%	80.64
	83.40%		886.21

observations (n) =	5	
Average ROR =	16.68%	= 83.40 / 5
Var = 886.21 / (5-1) =	221.552	= 886.21 / (5-1)
Standard Deviation=	14.88%	= SQRT 221.552

INFLATION, NOMINAL AND REAL RATES OF RETURN

Nominal Rate of Return (R) = 10%
Inflation (i) = 6.0%

$$\boxed{r = R - i}$$

Real Rate of Return (r) (Approximation) = 4.0%

$$\boxed{\text{Real Rate of Return (Exact)} = r = (R-i) / (1+i)}$$

Example
Invest in one-year CD for 8.0%. Inflation is 5.0%. Find the approximate and exact Real Rate of Return:

Approximate r = 8.00% - 5.00% = 3.00%
Exact r = (8.00% - 5.00%)/(1+5.00%) = 2.86%

EQUILIBRIUM NOMINAL RATE OF INTEREST

Fisher Equation .... R = r + E (i) ....Nominal rate ought to increase one for one with increase of expected inflation

ASSET ALLOCATION ACROSS RISKY AND RISK FREE PORTFOLIOS

Percentage across

Total Portfolio = **300,000**

Cash	90,000	30%
Stocks	210,000	70%
Total	300,000	

Stocks		of total Portfolio	of total Stocks
S&P 500 Index	113,400	37.800%	54.000%
Fidelity Invest	96,600	32.200%	46.000%
	210,000	70.000%	100.000%
Cash	90,000	30.000%	
	300,000	100.000%	

Portfolio Expected Return and Risk

+ Optimal Risky Portfolio (P)
+ Proportion of the Investment budget (Y) to be allocated to it.
+ The remaining portion (1-Y) is to be invested is the Risk-free Asset (rf)
+ Actual risk rate of return by rp on P by E(rp) and Standard Deviation σp
+ The rate on risk-free asset is denoted a rf

+ The portfolio return is E(rc)

E(rp) =	15%
σ =	22%
Rf =	7%
E(rp)-rf =	8%

Let's start with two extreme cases

1. If y=1 (all of the portfolio in the risk asset)

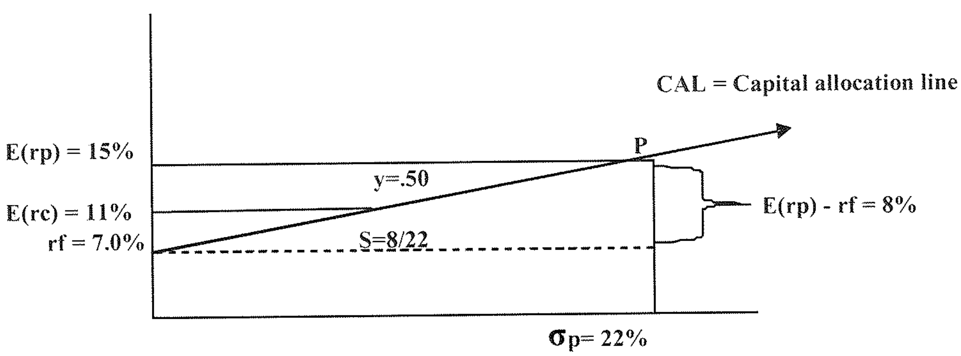
   E(rp) = 15%

   σp= 22%

2. If y=0 (none of the portfolio in the risk asset)

   rf= 7%

   σp= 0%



E(rc) = 1.5 * 7% + 0.5 * 15% = 11%

E(rc) - rf = y[E(rp) - rf]

$\sigma c = y \sigma p$

The Capital Allocation Line (CAL)

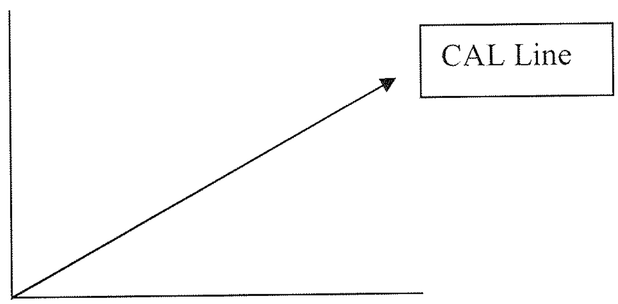
Different values of Y (risky portfolio)

The slope (s) of the CAL equals the increase in expected return that an investor can obtain per unit of additional standard deviation.

THE REWARD-TO-VOLATILITY RATIO (Sharpe Ratio)

	Exp. Return	Risk Premium	Standard Dev.	Sharpe Ratio
Portfolio P.	15%	8%	22%	8/22 = 0.36
Portfolio C.	11%	4%	11%	4/11 = 0.36

Plot on CAL the Sharpe Ratio is the same



Rf = 7% …. If the investor can borrow at (risk free) rate of rf=7.0%, then he/she can construct a complete portfolio that plot on the CAL line to the right of P where y>1

Example:

Originally have $300,000 – borrows additional $120,000 = $420,000 invested at y risk

This is a levered position in risky assets

y = 420,000 / 300,000 = 1.4x and 1-y= - 0.4, reflecting a short position in the risk-free assets – a borrowing position = 7.0%

The portfolio rate of return is

$E(rc) = 7 + (1.4 * 8) = 18.2\%$

Your income = \$63,000 (15% of \$420,000) and pay \$8,400 (7% of 120,000) interest

\$63,000 – 8,400 = 54,600

54,600 / 300,000 = 18.2%

<u>Sharpe Ratio:</u>

$\sigma i = 1.4 * 22 = 30.8$

$$\boxed{S = (E\ (ri) - rf) / \sigma i = (18.2 - 7.0) / 30.8 = 11.2 / 30.8 = 0.36}$$