LECTURE 11

OPTION VALUATION (Chapter 16)

INTRINSIC & TIME VALUES

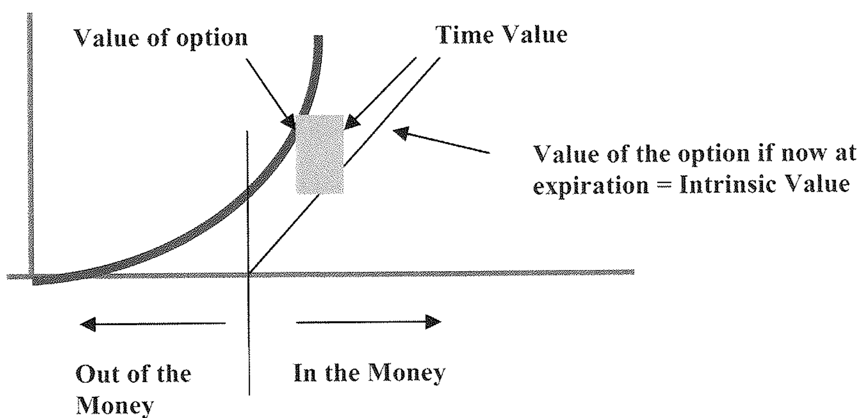
Consider a CALL option that is out of the money at the moment – which is stock below the exercise price – This does not mean that the value is Valueless.

There is always a chance that the stock will increase sufficiently by expiration date (or Zero value at Expiration day)

S – X = Intrinsic value

The difference between the Actual Call price and the value of the Intrinsic Value call Time Value of the option – It is the Volatility Value If not exercised the payoff cannot be less than Zero – As the price of the stock increases, the probability to be exercised is higher as it approaches the "adjusted" intrinsic value.

S – PV (x)



DETERMINING OF OPTION VALUE

Six factors that affect the value of Call option:

If the Value Increases		**The Value of the Call Option**
1. Stock Price	S	Increase
2. Exercise Price	X	Decrease
3. Volatility op the stock price	σ	Increase
4. The time to expiration	T	Increase
5. The interest rate	rf	Increase
56. Dividend Value of the stock	D	Decrease

VOLATILITY IMPACT

Value \$10 and \$50 = average \$30
Value \$20 and \$40 = average \$30

Both have the same average, but the volatility on the first one is much higher. Suppose the exercise price is \$30…. Option Payoff? With 1 in 5 probability 0.2.

High Volatility Scenario

Stock Price	10	20	30	40	50
Option Payoff	0	0	0	10	20

Low Volatility Scenario

Stock Price	20	25	30	35	40
Option Payoff	0	0	0	5	10

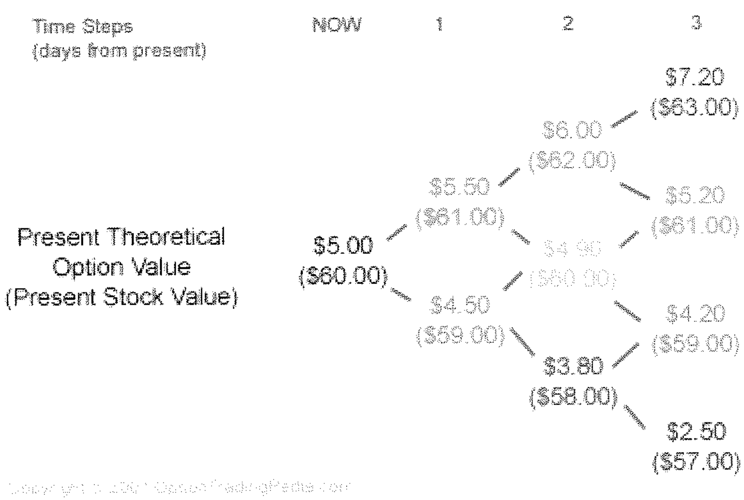
High Volatility Average = (0+0+0+10+20)/5 = 6
Low Volatility Average = (0+0+0+5+10)/5 = 3

So it doesn't matter if it's below \$30 (Zero value) – upside only volatility.

BINOMIAL OPTION PRICING MODEL

Binomial Option Pricing Model (BOPM) was invented by Cox-Rubinstein in 1979. It was originally invented as a tool to explain the Black-Scholes Model to Cox's students. However, it soon became apparent that the binomial model is a more accurate pricing model for American Style Options. The binomial model is thus named as it returns 2 possibilities at any given time. Therefore, instead of assuming that an option trader will hold an option contract all the way to expiration like in the Black-Scholes Model, it calculates the value of that trader exercising that option contract with every possible future up and down moves on its underlying asset, reflecting its effects on the present value of that option, thus giving a more accurate theoretical price of an American Style option.

The binomial model produces a binomial distribution of all the possible paths that a stock price could take during the life of the option. A binomial distribution, or simply known as a "Binomial Tree", assumes that a stock can only increase or decrease in price all the way until the option expires and then maps it out in a "tree". Here is a simplified version of a binomial distribution just for illustration purpose:



It then fills in the theoretical value of that stock's options at each time step from the very bottom of the binomial tree all the way to the top where the final, present, theoretical value of a stock option is arrived. Any adjustments to stock prices at an ex-dividend date or option prices as a result of early exercise of American options are worked into the calculations at each specific time step.

Advantage Of The Binomial Option Pricing Model

It can more accurately price American Style Options than the Black-Scholes Model as it takes into consideration the possibilities of early exercise and other factors like dividends.

Disadvantage Of The Binomial Option Pricing Model

As it is much more complex than the Black-Scholes Model; it is slow and not useful for calculating thousands of option prices quickly.

Example:

BINOMIAL OPTION PRICING

Probability of direction of the stock up or down 50/50

Parameters	Current Stock Price	Probability (p)	Stock x p	Call Option Payoff if Exercised	Net after Repayment of Loan	Relationship between Payoff and Profit (leverage)	Value of the Call Option
Current Price=	$ 100.00						
Up probability (u) =		1.2	$ 120.00	$ 10.00	$ 30.00	3.0x	$ 6.06
Down probability (d) =		0.9	$ 90.00	$ -	$ -		
Range =			$ 30.00				

Exercise Call Option =	$ 110.00
Exercise time =	1 year

Borrowing Parameters	
Interest Rate =	10%
Borrowed Amount (P) per share=	$ 81.82
Interest Amount per share =	$ 8.18
Total	$ 90.00

Sources of Investment	
Loan	$ 81.82
Cash (Equity)	$ 18.18
Total Sources	$ 100.00

Fully Hedged Portfolio

Stock Price	90	120
Obligations for 3 Calls	0	-30
Payoff	90	90

HEDGE RATIO

Using the same example

One share =	3.0x	Calls
Current Price=	$ 100.00	
Value d x 100 =	$ 90.00	
value u x 100	$ 120.00	
Range	$ 30.00	
Cu=	$ 10.00	
Cd =	$ -	
The Ratio (Range / Call payoff)	1/3	

Hedge Ratio Formula:
(H) = (Cu - Cd) / (uSo - dSo)

Cu and Cd call option going up or down
uSo, dSo are the stock prices in the two states

uSo =	$ 120.00
dSo =	$ 90.00
Exercise Price =	$ 110.00
Cu =	$ 10.00
Cd =	$ -
Stock Price Range =	$ 30.00
Option Price Range =	$ 10.00
Hedge Ratio (H) =	**1/3**

Portfolio Hedging

Share per option	1/3
Written option would have an end-of-year value with certainty =	$ 30.00
PV=	$ 27.27
Set Value of the hedged position equal to the PV of certain payoff =	$ 33.33
Call's Value	$ 6.06

Testing a Mispriced option against the Hedge Ratio

Assume Call option is mispriced at = $ 6.50

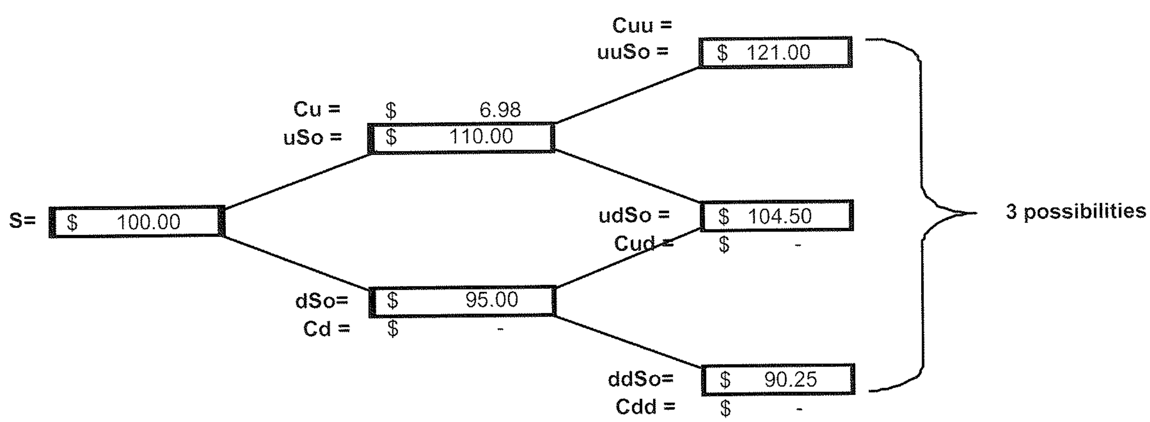
Using the Hedge ratio you get to the following strategy= 3.0x

Sequence	Strategy	Initial CF	At S1 = $ 90.00	At S1 = $ 120.00
FIRST	Write 3 Calls at cost =	$ 19.50	$ -	$ (30.00)
SECOND	Buy one share =	$(100.00)	$ 90.00	$ 120.00
THIRD	Borrow the difference at 10% =	$ 80.50	$ (88.55)	$ (88.55)
	Total	**$ -**	**$ 1.45**	**$ 1.45**
	Present Value=	$ -	$ 1.32	$ 1.32
	Per option profit		$ 0.44	$ 0.44

Note : The exact amount that the option is mispriced $6.50 - $6.06 = $ 0.44

GENERALIZING THE TWO-STATE APPROACH

Scenarios	p	Increase / Decrease by
u=	1.100x	10.0%
d=	0.950x	-5.0%



H = (Cuu - Cud) / (uuSo - udSo)

Example

Rf = 5% per six month
Exercise Price= $ 110.00
Value of option= ????

			Option Payoff
uSo $	110.00	Cu=	
uuSo = $	121.00	Cuu= $	11.00
udSo = $	104.50	Cud= $	-

H =	2/3

Thus the strategy is

			udSo =	uuSo=
			$ 104.50	$ 121.00
FIRST	Buy 2 shares at Price uSo =	$ 110.00	$ 209.00	$ 242.00
SECOND	Write 3 calls at price Cu		$ -	$ (33.00)
Total			$ 209.00	$ 209.00

The portfolio must have a current value equal to the present value of $209.

2x $110 - 3Cu = PV of 209.00 = $199.047

Cu = (209 - 220)/-3

PV=	199.047619

We solve for :

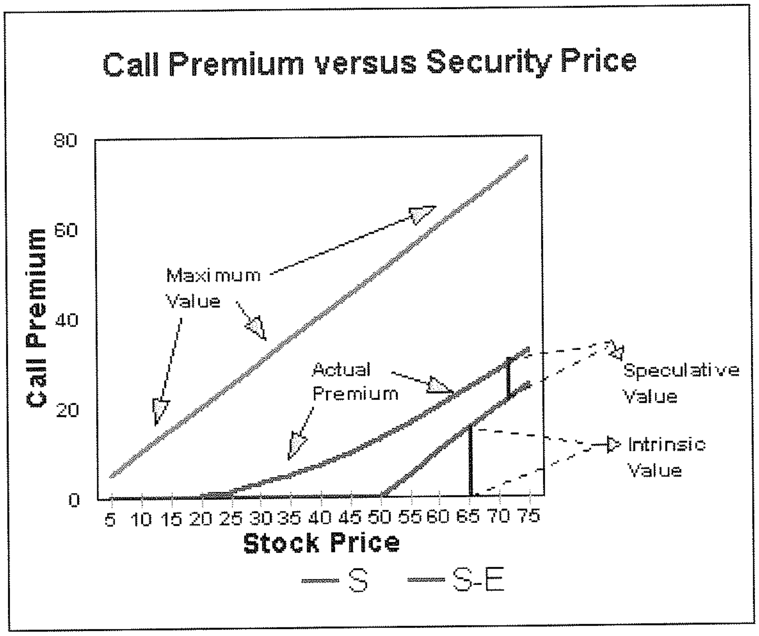
Cu = 6.984 (Cu = (220 - 199.047) /3)

Cd= $ - out of the money

C= $	4.43

Cu - Cd = $	6.98
uSo - dSo = $	15.00
H =	0.466x
H x 10 = $	4.656
PV = $	4.43

BLACK-SCHOLES OPTION PRICING MODEL



Black-Scholes Equation

Call Option = $SN(d_1) - Xe^{-rT}N(d_2)$

Where:

$$d_1 = \frac{\ln(\frac{S}{X}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The **Black–Scholes** model is a mathematical description of financial markets and derivative investment instruments. The model develops partial differential equations whose solution, the **Black–Scholes formula**, is widely used in the pricing of European-style options.

Black-Scholes Model - Definition
A mathematical formula designed to price an option as a function of certain variables-

generally stock price, striking price, volatility, time to expiration, dividends to be paid, and the current risk-free interest rate.

Black-Scholes Model - Introduction

The Black-Scholes model is a tool for equity options pricing. Prior to the development of the Black-Scholes Model, there was no standard options pricing method and nobody can put a fair price to charge for options. The Black-Scholes Model turned that guessing game into a mathematical science which helped develop the options market into the lucrative industry it is today. Options traders compare the prevailing option price in the exchange against the theoretical value derived by the Black-Scholes Model in order to determine if a particular option contract is over or under valued, hence assisting them in their options trading decision. The Black-Scholes Model was originally created for the pricing and hedging of European Call and Put options as the American Options market, the CBOE, started only 1 month before the creation of the Black-Scholes Model. The difference in the pricing of European options and American options is that options pricing of European options do not take into consideration the possibility of early exercising. American options therefore command a higher price than European options due to the flexibility to exercise the option at anytime. The classic Black-Scholes Model does not take this extra value into consideration in its calculations.

Black-Scholes Model Assumptions

There are several assumptions underlying the Black-Scholes model of calculating options pricing. The most significant is that volatility, a measure of how much a stock can be expected to move in the near-term, is a constant over time. The Black-Scholes model also assumes stocks move in a manner referred to as a random walk; at any given moment, they are as likely to move up as they are to move down. These assumptions are combined with the principle that options pricing should provide no immediate gain to either seller or buyer.

The exact 6 assumptions of the Black-Scholes Model are:

1. Stock pays no dividends

2. Option can only be exercised upon expiration

3. Market direction cannot be predicted, hence "Random Walk"

4. No commissions are charged in the transaction

5. Interest rates remain constant

6. Stock returns are normally distributed, thus volatility is constant over time

As you can see, the validity of many of these assumptions used by the Black-Scholes Model is questionable or invalid, resulting in theoretical values which are not always accurate. Hence, theoretical values derived from the Black-Scholes Model are only good as a guide for relative comparison and is not an exact indication to the over or under priced nature of a stock option.

Model assumptions

The Black–Scholes model of the market for a particular equity makes the following explicit assumptions:

- It is possible to borrow and lend cash at a known constant risk-free interest rate. This restriction has been removed in later extensions of the model.
- The price follows a Geometric Brownian motion with constant drift and volatility. It follows from this that the return is a Log-normal distribution. This often implies the validity of the efficient-market hypothesis.
- There are no transaction costs or taxes.
- The stock does not pay a dividend (see below for extensions to handle dividend payments).
- All securities are perfectly divisible (*i.e.* it is possible to buy any fraction of a share).
- There are no restrictions on short selling.
- There is no arbitrage opportunity
- Options use the European exercise terms, which dictate that options may only be exercised on the day of expiration.

From these conditions in the market for an equity (and for an option on the equity), the authors show that "it is possible to create a hedged position, consisting of a long position in the stock and a short position in [calls on the same stock], whose value will not depend on the price of the stock."[3]

Several of these assumptions of the original model have been removed in subsequent extensions of the model. Modern versions account for changing interest rates (Merton, 1976), transaction costs and taxes (Ingerson, 1976), and dividend payout (Merton, 1973).

The Black Scholes formula calculates the price of European put and call options. It can be obtained by solving the Black–Scholes partial differential equation.

The value of a call option in terms of the Black–Scholes parameters is:

$$C\ (S,t) = SN\ (d_1) - Xe^{-rt}\ N(d_2)$$

$$d_1 = [\ \ln (So/X) + (r + \sigma^2 / 2)\ (t)\ ] / [\ \sigma \sqrt{t})\ ]$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

The price of a put option is:

$$P\ (S,\ t) = Xe^{-rt)} - S+(SN(d_1) - Xe^{-rt}\ N(d_2)) = Xe^{-rt} - S+C\ (S,t)$$

For both, as above:

- N(•) is the cumulative distribution function of the standard normal distribution
- t is the time to maturity
- S is the spot price of the underlying asset
- X is the strike price
- r is the risk free rate (annual rate, expressed in terms of continuous compounding)
- σ is the volatility in the log-returns of the underlying

Interpretation

$N\ (d_1)$ and $N\ (d_2)$ are the probabilities of the option expiring in-the-money under the equivalent exponential martingale probability measure (numéraire = stock) and the equivalent martingale probability measure (numéraire = risk free asset), respectively. The equivalent martingale probability measure is also called the risk-neutral probability measure. Note that both of these are *probabilities* in a measure theoretic sense, and neither of these is the true probability of expiring in-the-money under the real probability measure. In order to calculate the probability under the real ("physical") probability measure, additional information is require

d—the drift term in the physical measure, or equivalently, the market price of risk.

Example

Suppose you want to value a call option under the following circumstances

Stock Price	$S_0 = 100$
Exercise Price	X=95
Interest Rate	r= .10
Dividend Yield	$\delta = 0$
Time to expiration	T = .25 (one-quarter year)
Standard Deviation	$\sigma = .50$

First calculate

d1 = [ln (100/95) + (.10-0 + $.5^2$/2).25] / [.5√.25] = .43

d2 = .43 - .5√.25 = .18

Next find N (d1) and d N (d2). The normal distribution function is tabulated and may be found in many statistics books. A table of N (d) is provided as Table 16.2 in the book page 521. The normal distribution function N (d) is also provided in any spreadsheet program. In Excel the function name is NORMSDIST, so using EXCEL (using interpolation for 43), we find that

N (.43) = .6664
N (.18) = .5714

Finally, remember that with dividends (δ) = o
$S_0 e^{-\delta T}$ = S0,

Thus the value of the call option is

$C = 100 \times .6664 - 95e^{-10x0.25} \times .5714$

=66.64 – 52.94 = $13.70

CHECK BLACK-SCHOLES SPREADSHEET ON THE WEB